

## Application of NSGA-II in Solving Multiobjective Optimal Power Flow

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*This paper is an application of NSGA-II for solving multiobjective optimal power flow problems in power systems. Objective functions considered in this work are conventional quadratic cost and emission along with highly non-linear features like cost curve with valve point loading and cubic emission function etc. In addition, more than two objectives are optimized simultaneously. The problem is formulated as mixed integer one with both continuous and discrete control variables. The performance of the proposed algorithm has been tested on three different IEEE test systems. Results for the test system-1 have been validated with the reported works. The comparison is done with the classical weighted sum method for IEEE-30 bus system and further experimentation is done on two other test cases such as IEEE-57 bus and IEEE-118 bus systems. The results demonstrate the effectiveness of the proposed approach for finding the Power System optimal solutions even when more than two conflicting objectives are considered simultaneously.*

**Keywords:** Multiobjective optimization, Optimal power flow, Nondominated sorting, Genetic algorithm.

### 1.0 INTRODUCTION

Optimal Power Flow (OPF) was first discussed by Carpentier in 1962 and gradually became one of the most important tool for power system studies over last few decades [1–4]. The OPF generally deals with finding new steady operating point of the complex power system where the minimization of generation costs, emissions, network losses, voltage deviations etc may be considered as objectives while the network must operate within its safe paradigm. Basically, the OPF is a highly nonlinear, nonconvex optimization problem consisting of both continuous and discrete control variables. Continuous control variables such as generators real power output, voltages etc and discrete control variables such as transformer tap settings, switched capacitors are adjusted to obtain the optimal operating point of the power system. OPF finds its application mainly in power

system operation and planning. Some of these are i) optimal dispatch of both real and reactive power ii) transmission expansion planning iii) corrective rescheduling for security enhancement of power system network etc. However, the objective of OPF in restructured electricity market has further added the maximization of social welfare in addition to minimizing generating units' emissions etc. Many classical optimization techniques have been tried to solve OPF problem, among them the most commonly used techniques are linear programming, quadratic programming, Newton based techniques, interior point methods etc. Although many of the classical techniques have excellent convergence characteristics and have been in use for years but they suffer from the drawbacks like convergence into local minima. The performance of some of the conventional optimization methods is found to be poor even when it has to handle both continuous and discrete

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control variables simultaneously [5]. The rapid developments of non conventional optimization techniques, such as evolutionary algorithms (EA), artificial intelligence etc and their successful use in power system research have been the motivating factors for many researchers for quite sometimes now [5–9].

With the increase in environmental awareness and its regulations, today's power system operation has become more complex with multiple goals. The use of evolutionary algorithm's (EAs) for solving optimization tasks with multiple objective has become very popular in the last few years. A multi-objective optimization problem (MOP) differs from a single objective optimization problem, because it contains several objectives that require optimization. For multi-objective problems with several (possibly conflicting) objectives, there is usually no single optimal solution. Therefore, the decision maker (DM) is required to select a compromised solution among many feasible solutions.

In some of the research works, multiobjective optimization problem was converted into a single objective problem by linear combination of different objectives as a weighted sum [10–13]. The important aspect of this method is that a set of non-inferior (or Pareto-optimal) solutions can be obtained by varying the weights. There is no rational basis for determining adequate weights and the objective function so formed may lose significance due to combining non commensurable objectives. Further, this method can not be used to find Pareto optimal solutions in problems having non-convex Pareto optimal front. To avoid this difficulty, the  $\epsilon$ -constraint method for multiobjective optimization was reported [14].

Over the past decade, a number of multiobjective evolutionary algorithms (MOEAs) have been suggested [15–20]. The potential of EAs for solving multi-objective optimization problem was hinted as early as the late 1960's by Rosenberg. But the implementation of MOEA was delayed until mid 1980's. Since EA work with a population of solutions, it is possible for EA to provide a diverge set of solutions for a multi-criterion environment.

Moreover, EAs are less susceptible to the shape or continuity of Pareto front.

Deb *et al.* [19] have proposed nondominated sorting GA (NSGA) for solving multiobjective problems. NSGA is a popular non-domination based genetic algorithm for multiobjective optimization. It is a very effective algorithm but has been criticized for its computational complexity, lack of elitism and for choosing the optimal parameter value for sharing parameter,  $\delta_{share}$ . An improved version of it, NSGA-II was developed by Deb *et al.* [20], which has a better sorting algorithm, incorporates elitism and no sharing parameter needs to be chosen and it has been demonstrated to be among the most efficient algorithm for multiobjective optimization on a number of benchmark problems. Recent works on power generation economics using multiobjective optimization methods [22,23] and the relevance of OPF as a utility tool in the new era of power system operation [24] have been the motivating factor in this work. Very few works are reported on the performance of NSGA-II on highly nonlinear multiobjective power system problems. In view of the above, the main objective of the present work is to develop a program based on NSGA-II and study its performance in solving multi-objective optimal power flow problem of various IEEE test systems.

## 2.0 PROBLEM FORMULATION

### 2.1 Mathematical Formulation

In the present work, the multi-objective optimal power flow has been formulated to minimize certain objectives subject to satisfying some network constraints. The multi-objective OPF problem can be written mathematically as follows:

Minimize,

$$F(x, u) = \begin{bmatrix} f_1(x, u) \\ \cdot \\ \cdot \\ f_k(x, u) \end{bmatrix} \quad k = 1, 2, \dots, \text{no of objectives.}$$

Subject to

$$g(x, u) = 0 \quad (1)$$

$$h(x, u) \leq 0$$

Where  $F(x,u)$  is the multi-valued objective function.  $x$  and  $u$  are vector of dependent variables and control variables respectively.  $g(x, u)$  and  $h(x, u)$  are equality and inequality equations associated with the problem.

For example, the dependent variables represent slack bus power, bus voltage angles, load bus voltage magnitudes etc whereas PV bus voltage magnitude, generated power, tap position of tap changers, shunt compensators etc are represented by the control variables.

## 2.2 Objective Function

### 2.2.1 Fuel Cost

The objective of fuel cost minimization is done by allocating best network settings that minimizes overall fuel cost function while satisfying other network constraints. The generator cost curve which is a function of power output, can be represented by the following equations without and with considering valve point loading respectively.

$$F(P) = \sum_{j=1}^{NG} (a_j + b_j P_j + c_j P_j^2) \quad (2)$$

$$F(P) = \sum_{j=1}^{NG} (a_j + b_j P_j + c_j P_j^2 + e_j \sin(f_j(P_{jmin} - P_j))) \quad (3)$$

### 2.2.2 Emission

The objective of fuel emission dispatch is done by allocating the best setting that minimizes the overall atmospheric emission. The total ton/hr emission  $E(P)$  of atmospheric pollutants such as sulphur oxides  $SO_2$  and nitrogen oxides  $NO_x$  caused by burning of fuel in thermal units can be expressed as:

$$E(P) = \sum_{j=1}^{NG} (\alpha_j + \beta_j P_j + \gamma_j P_j^3) \quad (4)$$

### 2.2.3 Power Loss

The objective of real power loss minimization is done by selecting the best combination of

variables, which minimizes the total real power loss of the network simultaneously satisfying all the network constraints. Mathematically it can be expressed as:

$$P_{loss} = \sum_{j=1}^m loss_j \quad (5)$$

### 2.2.4 Voltage Deviation

As voltage magnitude of load busses is one of the most important criterion to maintain power system stability and reliability issues, the sum of voltage deviations of all the load busses must be kept as small as possible. Mathematically it can be expressed as:

$$VD = \sum_{j=1}^{NL} |(V_j - V_j^{ref})| \quad (6)$$

The  $j$ th load bus reference voltage is generally considered as 1.0 p.u.

## 2.3 CONSTRAINTS

### 2.3.1 Power balance constraints

The total power generated by the units must be equal to the sum of total load demand and total real power loss in the transmission lines. Hence the equality constraint equations are:

$$P_{Gi} - P_{Di} - \left| V_i \left| \sum_{k=1}^{NB} V_k \left\{ \begin{array}{l} G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \\ \sin(\theta_i - \theta_k) \end{array} \right\} \right| \right| = 0 \quad (7)$$

$$Q_{Gi} - Q_{Di} - \left| V_i \left| \sum_{k=1}^{NB} V_k \left\{ \begin{array}{l} G_{ik} \sin(\theta_i - \theta_k) - \\ B_{ik} \cos(\theta_i - \theta_k) \end{array} \right\} \right| \right| = 0 \quad (8)$$

$i=1,2,..NB$

### 2.2.3 Generation capacity constraints

The real power output of generating units must be restricted within their respective lower and upper bounds (inequality constraints) as follows:

$$P_{Gj}^{\min} \leq P_{Gj} \leq P_{Gj}^{\max} \quad |j=1,2,..NG| \quad (9)$$

### 2.3.3 Other constraints

The other operational constraints involved in OPF problem are generator reactive power, voltage magnitudes of all generators, transformer tap positions, VAR compensator position, bus voltage magnitudes of all load buses etc. These constraints can be written as follows:

$$\begin{aligned}
 Q_{Gj}^{\min} \leq Q_{Gj} \leq Q_{Gj}^{\max} & \quad j=1,2,\dots,nPV \\
 |V_j^{\min}| \leq |V_j| \leq |V_j^{\max}| & \quad j=1,2,\dots,nPV \\
 |T_i^{\min}| \leq |T_i| \leq |V_i^{\max}| & \quad i=1,2,\dots,nT \\
 |Q_{ci}^{\min}| \leq |Q_{ci}| \leq |Q_{ci}^{\max}| & \quad i=1,2,\dots,nC \\
 |V_j^{\min}| \leq |V_j| \leq |V_j^{\max}| & \quad j=1,2,\dots,nPQ
 \end{aligned} \tag{10}$$

### 3.0 NSGA-II IMPLEMENTATION TO MOOPF PROBLEM

In principle, NSGA-II initializes by generating a random set of initial population of size  $p$ . Once the population is initialized, it is sorted based on non-domination into Pareto optimal fronts. For sorting the population based on their non-domination level, the fitness values are calculated by running the power flow program. In this paper, Newton Raphson power flow algorithm is used to minimize the real and reactive power mismatches. In order to reduce the computational complexity, the NSGA-II uses a special book-keeping strategy for a faster and efficient comparison of the solutions. Each solution is assigned a fitness (rank) equal to its non-domination level. The algorithm uses binary tournament selection, recombination and mutation operators to create offspring population of size  $p$ . Since elitism is important for the effectiveness of the search, the NSGA-II introduces elitism by including all population from current and previous generations and a combined population of size  $2p$  is formed. The population is sorted

again based on their non-domination and a new population of size  $p$  is selected. The diversity among the non-dominated solution is introduced by crowding distance comparison procedure, which is used during tournament selection and population reduction phase. The crowding distance measures how close an individual is to its neighbours and a larger crowding distance will result in better diversity in the population. From the selected population, a new offspring is created and the procedure is continued for subsequent generations until the stopping criteria specified by number of generations are met.

Recently, the application of NSGA-II has been reported by Baskar *et al.* for solving power system generation expansion planning problem [25] and reactive power dispatch problem [26] respectively. In both of the reported works, the performance of NSGA-II algorithm has been modified either by using controlled elitism or dynamic crowding distance etc. However, investigation can be carried out in finding the performance of simple NSGA-II in solving highly nonlinear multiobjective optimal power flow problem. Hence, application of NSGA-II has been reported in this paper formulating multi-objective optimal power flow for different sizes of electric power system network.

### 3.1 Initial Population

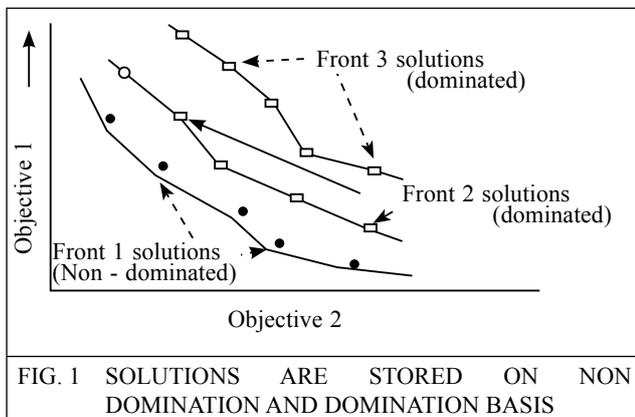
An initial population,  $P$  of size  $N \times n$ , where  $N$  is the number of individuals (chromosomes) and  $n$  is the number of control variables (continuous and discrete) is created choosing the values randomly from within their bounds. Generators real power output  $P_{ij}$  and voltage magnitudes  $|V_{ij}|$  are considered as  $T_{ij}$  continuous control variables whereas tap position of tapped transformers and switching position of shunt capacitors are considered as discrete control variables. Initially gene values of each individual are determined by setting its value randomly. For example, the real power output of  $j$ th generator of  $i$ th individual is found randomly such that its value lies between its lower and upper limits, i.e.,  $P_{ij} \sim U(P_{ij}^{\min}, P_{ij}^{\max})$ . The process is repeated to find the gene values of all other individuals (chromosomes).

### 3.2 Nondominated Sort

After the initial population  $P$  is generated, a non-dominated sorting of the population is done into different fronts. For each solution  $p$ , two attributes are found:

1. Domination count  $n_p$ , the number of solutions, which dominate the solution  $p$ , and
2.  $S_p$ , a set of solutions, which are dominated by solution  $p$ .

All solutions in the first non-dominated front will have their domination count  $n_p$  as zero. Now, for each solution  $p$  with  $n_p=0$ , each member ( $q \in S_p$ ) is found and its domination count is reduced by one. In doing so, if for any member the domination count becomes zero, it ( $q$ ) is included in a separate set  $Q$ . The members of this set  $Q$  belong to the second non-dominated front. This process continues until all fronts are identified. Figure 1 depicts the solutions classified into fronts after the method is applied.

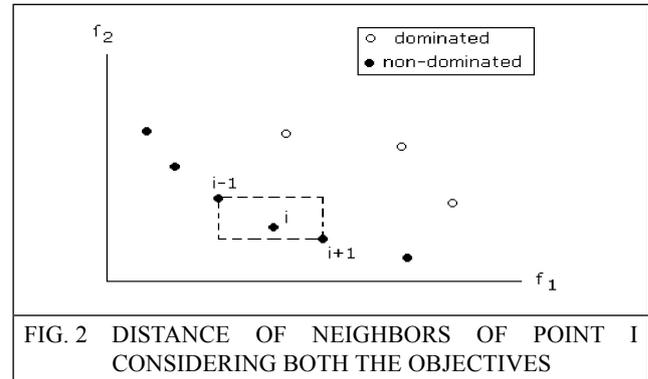


### 3.3 Density Estimation

To get an estimate of the density of solutions surrounding a particular solution in the population, the average distance of two points on either side of the point under consideration along each of the objectives is calculated.

A cuboid is formed by taking the nearest solutions on either side. The quantity  $i_{distance}$  serves as an estimate of the perimeter of the cuboid and is

called the *crowding distance*. Figure 2 depicts the crowding distance.



The computation of crowding-distance requires sorting the population according to each objective function value in ascending order of magnitude. Then, for each objective function, the solutions with smallest and largest function values are assigned an infinite distance value for the objective under consideration. All other solutions having function value intermediate between minimum and maximum values mentioned above are assigned a distance value equal to the absolute normalized difference in the function values of two adjacent solutions. This calculation is continued with other objective functions. The overall crowding-distance value for each solution is calculated as the sum of individual distance values corresponding to each objective. Each objective function is normalized before calculating the crowding distance.

### 3.4 Selection Algorithm

Non-dominated sorting based selection is used for selecting the population for the next generation. Initially, a combined population  $R_t = P_t \cup Q_t$  is formed, where  $P_t$  is the parent population and  $Q_t$  is the new population obtained after applying genetic operators. The population  $R_t$  is of size  $2N$ . The population  $R_t$  is sorted according to non-domination. Then crowding distance is calculated for each individual. Since  $N$  chromosomes are selected for next generation (i.e.  $P_{t+1}$ ) from  $2N$  chromosomes of combined population  $R_t$ , elitism is ensured. Now, solutions belonging to the non-dominated set  $F_1$  are the best solutions in the combined population and must be emphasized

more than any other solution during selection. While selecting  $N$  solutions from fronts starting with  $F_1$  the following conditions are considered within a front.

1. There may be more than one chromosome having zero crowding distance and/or
2. Different solutions having a crowding distance less than  $\epsilon$ , a threshold value.

Condition 1 is an indication of duplicate chromosomes and condition 2 where chromosomes are having a crowding distance less than  $\epsilon$  is an indication of close proximity of solutions which, if accepted, will result into cluster of solutions which is undesired. Algorithm selects only one solution in case of duplicate chromosomes and rejects chromosomes having crowding distance less than  $\epsilon$ . If the number of solutions selected from front  $F_1$  is less than  $N$ , the remaining ( $\gamma$ ) members of the population  $P_{t+1}$  are chosen from subsequent nondominated fronts in the order of their ranking. Thus, solutions from the set  $F_2$  are chosen next, followed by solutions from the set  $F_3$  and so on till  $N$  number of solutions is selected. While selecting, the solutions are accepted from best to worst front ( $F_1, F_2, \dots$ ), but with non acceptance of all solutions of any particular front, there is a chance of not getting all  $N$  chromosomes even from all the fronts (having  $2N$  chromosomes). In such case, population is filled up by duplicating the accepted solutions. The new population  $P_{t+1}$  of size  $N$  is now used for selection, crossover, and mutation to create a new population  $Q_{t+1}$  of size  $N$ .

### 3.5 Creation of Offspring

Here, real-coded GA (SBX-Simulated Binary Crossover) is used for crossover and Polynomial mutation is used [21] for mutation for offspring generation. The SBX operator works with two parent solutions and creates two offspring. The two offspring created are symmetric about the parent solutions are as follows:

$$c_{1,k} = \frac{1}{2} \left[ (1 - \beta_k) p_{1,k} + (1 + \beta_k) p_{2,k} \right] \quad (11)$$

$$c_{2,x} = \frac{1}{2} \left[ (1 + \beta_k) p_{1,k} + (1 - \beta_k) p_{2,k} \right] \quad (12)$$

where  $c_{i,k}$  is  $i$ th child's  $k$ th gene.  $\beta_k$  is the is a random number as below:

$$\beta(u) = (2u)^{\frac{1}{\eta+1}} \text{ or}$$

$$\beta(u) = \frac{1}{\left[ 2(1-u) \right]^{\frac{1}{\eta+1}}}$$

where  $u$  is a uniform random number between limits (0,1). Any one of  $\beta(u)$  is selected randomly.

**Polynomial Mutation:** The probability of creating a solution near to the parent is higher than the probability of creating one distant from it. The shape of the probability distribution is directly controlled by an external parameter  $\eta_m$  and the distribution remains unchanged throughout the iterations.

$$c_k = p_k + (p_k^u - p_k^l) \delta_k \quad (13)$$

where  $c_k$  is the child and  $p_k$  is the parent with  $p_k^u$  being the upper bound and  $p_k^l$  is the lower bound and  $\delta_k$  is small variation which is calculated from a polynomial distribution by using

$$\delta_k = (2r_k)^{\frac{1}{\eta_m+1}} \quad \text{if } r_k < 0.5$$

$$\delta_k = 1 - \left[ 2(1-r_k) \right]^{\frac{1}{\eta_m+1}} \quad \text{if } r_k \geq 0.5$$

where  $r_k$  is a uniform random number between limits (0,1).

### 3.6 Stopping Rule

The iterative procedure of generating new trials by selecting those with minimum function values from the competing pool is terminated when there is no significant improvement in the solution. It can also be terminated when a given maximum number of generations (iterations) is reached. In the present work, the latter method

is followed. However, the value of maximum number of generations is decided after a number of trial runs.

#### 4.0 SIMULATION RESULTS

The algorithm is implemented on MATLAB, version 7.2 for solving Multiobjective Optimal Power Flow (MOOPF) and is experimented on various IEEE test cases with modifications. The non-dominated solutions corresponding to the Pareto front is obtained for each test case. The control variables were considered as both continuous and discrete for all the test cases. To verify the effectiveness of the proposed approach in solving MOOPF, results of the extreme cases (minimum cost, minimum emission, minimum real power loss, minimum voltage deviations etc.) are presented after executing 25 trial runs for each test case. For comparison, the results obtained with IEEE-30 bus system with the proposed algorithm has been verified with those reported in [5]. Further, the same results have been compared with classical weighted sum method for the first two cases of IEEE-30 bus system. The application of the proposed algorithm has been extended for IEEE-57 bus and IEEE-118 bus system. The generator characteristics as well as cost and emission data for IEEE-57 bus and IEEE-118 bus system is shown in Appendix. The GA parameters used in the simulation work of this paper is stated below.

Population size	: 100
Crossover overtype	: simulated binary crossover
Mutation	: polynomial mutation
Crossover probability	: 0.9
Mutation probability	: $1/n$ , where n is the number of decision variables
Maximum generation	: 200

#### 4.1 IEEE 30 Bus System

The network, load, generation and emission data were taken from ref. [5]. The system is having six generators (at buses 1, 2, 13, 22, 23, 27), four tap changing transformers

(between buses 6–9, 6–10, 4–12, 27–28) and two switchable capacitor banks (at buses 5 and 24). The operating range of all transformers is set between 0.9–1.05 with a discrete step size of 0.01 and the range of capacitor banks are considered between 0–40 MVar with a step size of 1. Three different cases have been studied for the same test system.

##### 4.1.1 Case-1

In this case, quadratic cost and emission have been considered as objectives to minimize while all other network operational constraints were taken into consideration. The values of the control variables for minimum cost and minimum emission have been presented and compared between GA based classical weighted sum method with NSGA-II in Table 1. It has been revealed that the minimum cost achieved using NSGA-II is 574.83 \$/h as compared to the value 575.17 \$/h achieved with GA based weighted sum approach. Similarly, the minimum emission obtained with NSGA-II is 284.23 ton/h as compared to the value 284.26 ton/h obtained with GA based weighted sum approach. The non-dominated solutions corresponding to the Pareto front obtained using NSGA-II has been shown in Figure 3. These results again have been verified with those reported in ref. [5].

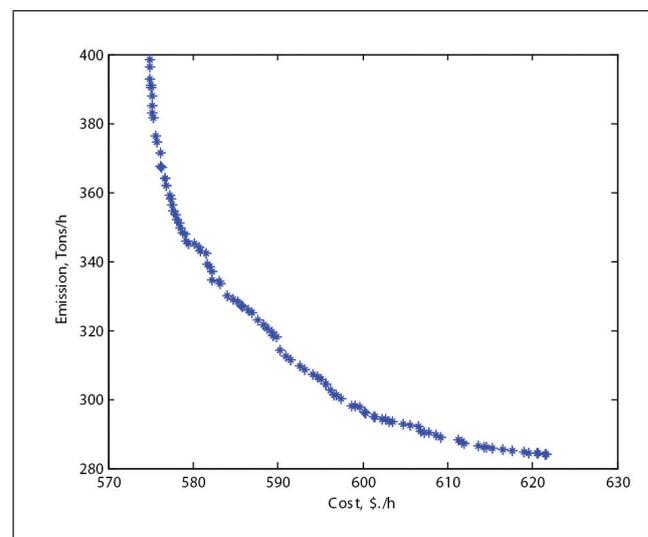


FIG. 3 QUADRATIC COST VS QUADRATIC EMISSION

TABLE 1				
QUADRATIC COST AND QUADRATIC EMISSION AS OBJECTIVES FOR IEEE 30 BUS SYSTEM				
Variables	Classical weighted sum with GA		NSGA-II	
	For minimum cost	For minimum emission	For minimum cost	For minimum emission
$P_{G1}$	44.0	23.099	43.723	24.657
$P_{G2}$	57.42	26.32	57.116	28.958
$P_{G13}$	22.95	35.45	22.592	32.267
$P_{G22}$	32.0	46.22	35.37	35.541
$P_{G23}$	18.25	29.705	16.455	29.994
$P_{G27}$	17	31	16.449	40
$V_1$	1.0	1.0	1.0	1.0
$V_2$	0.99	1.0035	0.99881	0.99884
$V_{13}$	1.027	0.9587	1.05	1.05
$V_{22}$	1.0106	0.9910	1.0496	1.0457
$V_{23}$	1.011	0.999	1.0445	1.05
$V_{27}$	1.0233	1.0132	1.0472	1.05
$T_{6-9}$	0.98	0.97	0.9	0.93
$T_{6-10}$	0.97	1.04	0.99	0.97
$T_{4-12}$	1.01	1.01	0.92	0.95
$T_{27-28}$	0.95	0.97	0.97	0.97
$Q_{C5}$	10	19	9	22
$Q_{C24}$	13	6	12	19
<b>Objectives</b>	<b>575.17 \$/h</b>	<b>284.26 ton/h</b>	<b>574.83 \$/h</b>	<b>284.23 ton/h</b>
Ploss	2.42 MW	2.590 MW	2.505 MW	2.217 MW

#### 4.1.2 Case-2

In this case, non smooth fuel cost function is considered with valve point loading effects and the emission is kept same as earlier. The values of the control variables for the extreme cases are reported in Table 2 and the non-dominated solutions corresponding to the Pareto front obtained using NSGA-II is depicted in Figure 4. In this case, the minimum generation cost has been found as 618.63 \$/h with NSGA-II against 691.88 \$/h with GA based weighted sum approach. The same result is again found to be in agreement with those reported in ref. [5]. The value of the second objective i.e. emission is found to be less against weighted sum approach.

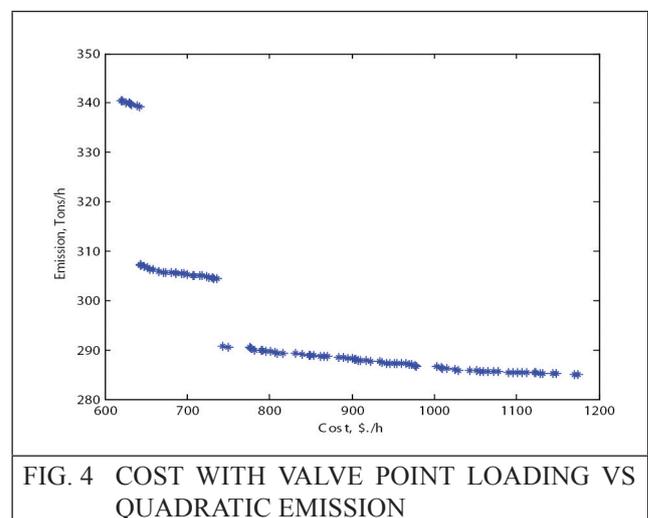


FIG. 4 COST WITH VALVE POINT LOADING VS QUADRATIC EMISSION

TABLE 2				
COST WITH VALVE POINT LOADING AND QUADRATIC EMISSION AS OBJECTIVES FOR IEEE 30 BUS SYSTEM				
Variables	Classical weighted sum with GA		NSGA-II	
	For minimum cost	For minimum emission	For minimum cost	For minimum emission
$P_{G1}$	41.424	24.50	41.434	25.187
$P_{G2}$	66.936	30.38	38.551	28.727
$P_{G13}$	24.969	35.49	24.958	32.17
$P_{G22}$	13.6512	44.46	31.443	35.299
$P_{G23}$	18.98	29.59	19.007	30
$P_{G27}$	27.97	27.21	38.867	40
$V_1$	1.0	1.0	1.0	1.0
$V_2$	1.0214	0.9957	0.99264	0.99936
$V_{13}$	0.9869	1.0306	0.996	1.0499
$V_{22}$	0.9917	1.079	1.05	1.049
$V_{23}$	1.021	1.0173	1.0496	1.05
$V_{27}$	0.99	1.0095	1.0494	1.0493
$T_{6-9}$	0.95	1.0	0.9	0.9
$T_{6-10}$	0.99	0.97	0.9	0.97
$T_{4-12}$	0.93	0.96	0.9	0.95
$T_{27-28}$	0.94	1.01	0.94	1
$Q_{C5}$	34	8	36	20
$Q_{C24}$	38	7	19	8
<b>Objectives</b>	<b>691.88 \$/h</b>	<b>284.45 ton/h</b>	<b>618.63 \$/h</b>	<b>284.1 ton/h</b>
Ploss	4.73 MW	2.43 MW	3.06 MW	2.183 MW

### 4.1.3 Case-3

In this case, along with quadratic cost function, real power loss and sum of voltage deviations are considered as objectives to minimize while satisfying all constraints. The values of the control variables for minimum fuel cost, minimum real power loss and minimum sum of voltage deviations have been reported in Table 3 and the non dominated solutions have been presented in Figure 5. The minimum value of fuel cost is found to be 578.93 \$/h after executing 25 trial runs and

the corresponding values of the other objectives may also be easily calculated by executing a full ac power flow program with the reported values of the control variables from Table 3.

Similarly, for the reported minimum power loss or voltage deviations, the other objectives (possibly conflicting) may be easily found by running a power flow program with the specified values of variables.

TABLE 3			
QUADRATIC COST, POWER LOSS AND VOLTAGE DEVIATIONS AS OBJECTIVE FOR IEEE 30 BUS SYSTEM			
Variables	For minimum fuel cost	For minimum power loss	For minimum voltage deviation
$P_{G1}$	43.28	28.631	30.652
$P_{G2}$	57.69	44.758	75.694
$P_{G13}$	22.114	37.808	19.169
$P_{G22}$	31.265	45.615	29.736
$P_{G23}$	12.995	11.275	10.061
$P_{G27}$	25.5	23.251	27.562
$V_1$	1.0	1.0	1.0
$V_2$	1.0066	0.9947	1.0267
$V_{13}$	1.0459	1.0284	1.0154
$V_{22}$	1.0364	1.0105	1.0099
$V_{23}$	1.017	1.0009	1.0473
$V_{27}$	1.0481	1.028	1.0012
$T_{6-9}$	0.95	0.99	1.0
$T_{6-10}$	0.90	0.92	0.92
$T_{4-12}$	0.98	0.99	1.0
$T_{27-28}$	0.9	0.99	1.04
$Q_{C5}$	26	31	17
$Q_{C24}$	11	8	4
<b>Objectives</b>	<b>578.93 \$/h</b>	<b>2.1376 MW</b>	<b>0.2043 p.u</b>

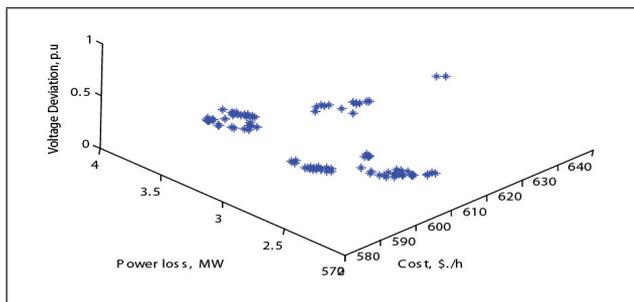


FIG. 5 QUADRATIC COST VS POWER LOSS VS VOLTAGE DEVIATION

#### 4.2 IEEE-57 bus and IEEE-118 bus System

In order to verify the effectiveness of the algorithm in solving the similar problems of larger dimensions, MOOPF programs were developed

for IEEE-57 and IEEE-118 bus system. There are 33 control variables considered for IEEE-57 bus system which consists of seven generators, seventeen tap changers and three shunt capacitors. For IEEE-118 bus system a total of 130 numbers of control variables considered in this MOOPF problem consisting of fifty four generators, nine tap changers and fourteen shunt capacitors. The control variables are modelled as continuous and discrete variables for both the test systems. The operating range of all transformers is set between 0.9–1.05 with a discrete step size of 0.01 for both the test systems. The ranges of switchable capacitors are assumed between 0–60 MVar and 0–40 MVar with a step size of 1 for IEEE-57 bus and IEEE-118 bus system respectively. The algorithm finds the optimal solution by

TABLE 4				
QUADRATIC COST AND CUBIC EMISSION AS OBJECTIVES				
Test system	Minimum cost	Mean cost	Minimum emission	Mean emission
57 bus	26726 \$/h.	26886 \$/h.	6.95 ton/h.	6.96 ton/h.
118 bus	142360 \$/h	151180 \$/h.	23.5 ton/h.	23.69 ton/h.

TABLE 5				
COST WITH VALVE POINT LOADING AND CUBIC EMISSION AS OBJECTIVES				
Test system	Minimum cost	Mean cost	Minimum emission	Mean emission
57 bus	27400 \$/h.	27560 \$/h.	6.95 ton/h	6.96 ton/h.
118 bus	149570 \$/h.	163546 \$/h.	23.566 ton/h	23.8 ton/h.

TABLE 6						
COST WITH VALVE POINT LOADING, POWER LOSS AND VOLTAGE DEVIATIONS AS OBJECTIVES FOR IEEE 57 BUS SYSTEM						
Objective	Minimum			Mean		
	Cost \$/h	Power loss in MW	Voltage deviation	Cost \$/h	Power loss in MW	Voltage deviation in p.u
Fuel cost	27709	20.85	3.143	28927	14.48	1.3207
Power loss	37571	12.62	4.42			
Voltage deviations	33747	26.83	1.0381			

TABLE 7						
COST WITH VALVE POINT LOADING, POWER LOSS AND VOLTAGE DEVIATIONS AS OBJECTIVES FOR IEEE 118 BUS SYSTEM						
Objective	Minimum			Mean		
	Cost \$/h	Power loss in MW	Voltage deviation	Cost \$/h	Power loss in MW	Voltage deviation in p.u
Fuel cost	161300	115.69	1.0987	166180	76.713	1.02809
Power loss	183290	71.5	1.3268			
Voltage deviations	165410	103.34	0.94708			

adjusting the control variables following NSGA-II optimization algorithm. Many trial runs were executed to find extreme as well as mean values of the objective functions considered. Results for only the objective functions are presented below from Table 4 through Table 7 for both the test systems. Values of the control variables are not presented because of space limit.

## 5.0 CONCLUSION

In this paper, a multi-objective optimal power flow program based on NSGA-II has been developed in Matlab and its performance has

been investigated on three different IEEE test cases. The problem has been formulated as true multi-objective optimization problem with competing and non-commensurable objectives. The ability of the algorithm has been tested with the presence of both continuous and discrete control variables. For validation, results have been well compared and found to be competitive for IEEE-30 bus system. The approach is further tested in solving problem of larger size e.g on test cases like IEEE-57 and IEEE-118 bus systems. The results obtained demonstrate that the algorithm is well competent and efficient in solving highly non linear mixed-integer multi-objective optimization (OPF) problems.

## APPENDIX

**Generator Characteristic of IEEE-57 bus System**

A	B	C	E	F	A	B	Γ
647.81	6.79	0.075	300	0.035	0.278	0.0050	7.64E-10
1055.1	3.33	0.521	120	0.077	0.0266	0.0047	2.21E-9
895.2	5.88	0.452	120	0.077	0.0203	0.0046	8.37E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.0047	2.21E-9
1728.3	9.15	0.007	300	0.035	0.1349	0.0052	7.89E-9
1055.1	3.33	0.521	120	0.077	0.0266	0.0047	2.21E-9
654.69	12.8	0.005	200	0.042	0.155	0.0045	3.2E-9
Gen	1	2	3	4	5	6	7
Pmax	575	100	140	100	550	100	410

**Generator Characteristic of IEEE-118 bus System**

A	B	C	E	F	A	B	Γ
1055.1	3.33	0.521	120	0.077	0.026	0.0047	2.21E-9
1055.1	3.33	0.521	120	0.077	0.026	0.0047	2.21E-9
1055.1	3.33	0.521	120	0.077	0.026	0.0047	2.21E-9
1055.1	3.33	0.521	120	0.077	0.026	0.0047	2.21E-9
1728.3	9.15	0.0076	300	0.035	0.134	0.0052	7.89E-10
309.54	7.07	0.02028	100	0.084	0.026	0.0047	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.026	0.0047	2.21E-9

1055.1	3.33	0.5212	120	0.077	0.026	0.0047	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.026	0.0047	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.026	0.0047	2.21E-9
635.20	12.9	0.00515	200	0.042	0.154	0.0035	3.17E-9
654.69	12.8	0.00569	200	0.042	0.155	0.0045	3.28E-9
1055.1	3.33	0.5212	120	0.077	0.026	0.0047	2.21E-9
987.7	6.5	0.52	120	0.077	0.026	0.0047	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.026	.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
955.5	4.5	0.52	100	0.084	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
310.5	8.05	0.026	100	0.084	0.0203	0.00512	8.37E-9
645.2	12.8	0.0052	200	0.042	0.1437	0.0046	3.18E-9
1055.1	3.33	0.5212	120	0.077	0.0204	0.0048	8.37E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
222.92	6.45	0.002	150	0.063	0.0108	0.00516	8.72E-9
107.87	8.63	0.0017	200	0.042	0.0204	0.0067	8.72E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
<b>a</b>	<b>b</b>	<b>c</b>	<b>e</b>	<b>f</b>	<b><math>\alpha</math></b>	<b><math>\beta</math></b>	<b><math>\gamma</math></b>
794.53	6.66	0.084	300	0.035	0.135	0.00252	7.89E-10
794.53	6.66	0.084	300	0.035	0.135	0.00525	7.89E-10
647.8	7.57	0.045	300	0.035	0.28	0.0052	7.89E-10
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
647.81	6.79	0.075	300	0.035	0.278	0.00503	7.64E-10
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
647.8	7.97	0.055	300	0.035	0.274	0.0051	7.7E-10
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9

1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
635.2	12.9	0.003	200	0.042	0.1545	0.0036	3.17E-9
895.2	5.88	0.452	120	0.077	0.0203	0.00461	8.37E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
945.5	6.3	0.452	120	0.077	0.02	0.0047	8.38E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9
1055.1	3.33	0.5212	120	0.077	0.0266	0.00476	2.21E-9

#### Generator limits of IEEE-118 bus System

Unit	Pmax	Unit	Pmax	Unit	Pmax	Unit	Pmax
1	100	15	100	29	492	43	100
2	100	16	100	30	805.2	44	100
3	100	17	100	31	100	45	352
4	100	18	100	32	100	46	140
5	550	19	100	33	100	47	100
6	125	20	119	34	100	48	100
7	100	21	304	35	100	49	100
8	100	22	148	36	100	50	100
9	100	23	100	37	577	51	136
10	100	24	100	38	100	52	100
11	320	25	225	39	104	53	100
12	414	26	260	40	707	54	100
13	100	27	100	41	100		
14	107	28	491	42	100		

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