Modelling and Analysis of 3-Phase Inverter for Grid Connected Solar PV system with Harmonic Compensation

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Renewable Energy Sources (RES) are getting a great attention due to ever increasing power demand, environmental issues, and thrust toward sustainable development. The power produced from the RES like solar, wind etc. is used by the stand alone system or the feed into the electric grid. In this paper a 3-phase Voltage Source Inverter (VSI) is modeled and analyzed for grid connected Solar Photovoltaic (SPV) system. Mathematical model of three phase VSI and LC filter circuits are derived using state space analysis and Sinusoidal Pulse Width Modulation (SPWM) technique is proposed for 3-phase VSI. Harmonic elimination is done in VSI using by Pulse Width Modulation (PWM) technique by solving the non-linear equations that are used to determine switching angles of an inverter. The switching angle plays an important role to produce desired output by eliminating selected harmonics. The VSI is simulated using MATLAB/Simulink platform and harmonic elimination is done by considering bipolar switching case of the inverter. Various simulation results are presented to demonstrate the operation of the inverter. The harmonic spectrum analysis is carried out and discussed for the proposed harmonic elimination method

Keywords: Pulse width modulation, Renewable energy sources, Voltage source Inverter

1.0 INTRODUCTION

Around the world, the penetration level of RES is increasing day by day due to increasing power demand and environmental issues. Different RES like solar, wind and geo-thermal etc. are used as source for electric power production. Solar energy is one of the important sources due to no pollution and less maintenance based power generation. The power produced from the solar is supplied to either standalone or grid connected systems. The best way is to feed the power into the electric grid. The electric power from the solar photovoltaic (SPV) is transferred to the grid by using Power Electronic Converters (PEC). Power converters are used in these systems to supply good quality of power to the consumer loads that also in a way increases the efficiency of the entire system[1]. So the application of SPV systems is increasing due to the use of PEC for efficient utilization of these systems. Due to the variations in the solar irradiation levels and operating temperature, the voltage and current of the SPV system are varying. So in order to get the maximum power from the solar panels, various Maximum Power Point Tracking (MPPT) techniques are employed to the solar PV systems. The power from the solar PV systems is DC. The PEC which converts DC power to AC power is called inverter. Voltage Source Inverter (VSI) is used to convert a dc voltage into periodic ac voltage with grid frequency such that power injected into the grid follows regular standard IEEE 1547. A low pass filter circuit is used to

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filter out harmonics produced by VSI. The output of the inverter voltage and currents should have less harmonics and good power quality. Only with satisfactory control performance of each individual unit, connecting two or more inverters in parallel or connecting one or more inverters to the power systems could be possible that also involves active and reactive power control under various load and operating characteristics. The solar PV system should be able to operate under grid connected mode or islanding mode. In case of grid connected mode the system should have low Total Harmonic Distortion (THD) and also should have strong control of active and reactive power under different operating conditions [2].

In this paper a 3-phase VSI is modeled and analyzed for grid connected solar PV system. Mathematical modeling of 3-phase VSI with LC filter is given and Sinusoidal Pulse Width Modulation (SPWM) technique is proposed for 3-phase VSI. The important work of this paper is elimination of harmonic in the VSI using Pulse Width Modulation (PWM) technique, by solving the non-linear equations. Equations are used to determine switching angles of an inverter. The switching angle plays an important role to produce desired output by eliminating selected harmonics. Inverter's bipolar switching case is used for harmonic elimination and also for control purpose. The paper is organized as follows. Section 2 deals with basic structure of grid connected solar PV systems and section 3 gives mathematical model of 3-phase VSI with LC filter and SPWM technique is also explained. In section 4 harmonic elimination method is analyzed. Section 5 gives simulation results to demonstrate the operation of the inverter and harmonic elimination method followed by conclusion in the last section.

2.0 GRID CONNECTED SOLAR PV SYSTEM

The general grid connected solar PV system is shown in Figure 1. The power from the solar PV system is dc and step up and regulation of input dc voltage is done by using converter and this converter is also used for MPPT. The power from SPV is converted from dc voltage into ac voltage for further usage and grid integration.

2.1 Modeling of Solar PV System

Solar cells are basically p-n junction diode and these cells are used to produce electricity directly by converting solar energy into electrical energy. Generally PV cells are connected in series and parallel in order to produce required voltage and current levels. The solar PV cell is a non-linear device and can be represented by a current source in parallel with diode as shown in Figure 1. The characteristics of the equivalent solar cell are given in 1.

$$I_{PV} = N_P * I_{Ph} - N_P * I_0 [\exp\{\frac{q * (V_{PV} + I_{PV}R_s)}{N_s AkT}\} - 1]$$
....(1)

Where I_{pv} is the PV array output current, V_{pv} is the PV array output voltage, I_{ph} is module photo current, R_s is the series resistance, k is the Boltzmann constant (138e-²³J/K), A is the ideal factor, N_s is the number of cells in series and N_p is number of cells in parallel. T is the operating Temperature [2-3].

The equation (1) is simulated using MATLAB/ Simulink and P-V and I-V characteristics are obtained. The operating curves shows that solar PV output power is the function of solar irradiation. P-V and I-V characteristic of a PV array with radiation of 1kW/m² is shown in Figure 2.





2.2 3-PHASE VSI

A 3-phase VSI is used to convert DC voltage into AC voltage and feeds power to consumer loads and utility grid. The 3-phase inverters are used in grid connected SPV systems. A 3-phase inverter is a six step bridge inverter and it uses a minimum of six devices. The transistor family of devices is now very widely used in inverter circuits and the use of IGBT in three-phase inverter is on the rise presently. A large capacitor connected at the input terminals tends to make the input terminals to make the input dc voltage constant. This capacitor also suppresses the harmonics fed back to the source. In inverter terminology, a step is defined as a change in the firing from one IGBT to the next IGBT in proper sequence. For one cycle of 360, each step would be of 60 intervals for a six step inverter. This means that the IGBT would be gated at regular intervals of a six step inverter. There are two possible patterns of gating the switches. In one pattern, each switch conducts for 180 and in the other each switches conducts for 120. But in both these patterns gating signals are applied and removed at 60 intervals of the output voltage [4].

3.0 MATHEMATICAL MODELING OF LC FILTER CIRCUIT

The mathematical model of LC filter circuit has been derived using state space analysis. LC output filter circuit for voltage and current equations is shown in Figure 4. Kirchhoff's current law is applied to the nodes a,b,c of the circuit shown in Figure 3.



At node a,

$$i_{iA} + i_{ca} = i_{ab} + i_{LA} \Longrightarrow i_{iA} + C_f \frac{dV_{LCA}}{dt} = C_f \frac{dV_{LBC}}{dt} + i_{LB} \dots (2)$$

At node b,

$$i_{iB} + i_{ab} = i_{bc} + i_{LB} \Longrightarrow i_{iB} + C_f \frac{dV_{LAB}}{dt} = C_f \frac{dV_{LBC}}{dt} + i_{LB} \dots (3)$$

1 . .

At node c,

$$i_{iC} + i_{bc} = i_{ca} + i_{LC} \Longrightarrow i_{iC} + C_f \frac{dV_{LBC}}{dt} = C_f \frac{dV_{LCA}}{dt} + i_{LC} \dots (4)$$

Now to make state space equations, subtract (3) from (2),

$$i_{iA} - i_{iB} + C_f \left(\frac{dV_{LCA}}{dt} - \frac{dV_{LAB}}{dt}\right) = C_f \left(\frac{dV_{LAB}}{dt} - \frac{dV_{LBC}}{dt}\right) + i_{LA} - i_{LB}$$

$$\Rightarrow C_f \left(\frac{dV_{LCA}}{dt} - 2\frac{dV_{LAB}}{dt} + \frac{dV_{LBC}}{dt}\right) = -i_{iA} + i_{iB} + i_{LA} - i_{LB}$$
....(5)

Similarly, subtracting (4) from (3),

$$i_{iB} - i_{iC} + C_f \left(\frac{dV_{LAB}}{dt} - \frac{dV_{LBC}}{dt}\right) = C_f \left(\frac{dV_{LBC}}{dt} - \frac{dV_{LCA}}{dt}\right) + i_{LB} - i_{LC}$$

$$\Rightarrow C_f \left(\frac{dV_{LAB}}{dt} - 2\frac{dV_{LBC}}{dt} + \frac{dV_{LCA}}{dt}\right) = -i_{iB} + i_{iC} + i_{LB} - i_{LC}$$
....(6)

And subtracting (2) from (4),

$$i_{iC} - i_{iA} + C_f \left(\frac{dV_{LBC}}{dt} - \frac{dV_{LCA}}{dt}\right) = C_f \left(\frac{dV_{LCA}}{dt} - \frac{dV_{LAB}}{dt}\right) + i_{LC} - i_{LA}$$

$$\Rightarrow C_f \left(\frac{dV_{LBC}}{dt} - 2\frac{dV_{LCA}}{dt} + \frac{dV_{LAB}}{dt}\right) = -i_{iC} + i_{iA} + i_{LC} - i_{LA}$$
....(7)

The algebraic sum of line to line load is equal to zero. So,

$$V_{LAB} + V_{LBC} + V_{LCA} = 0 \qquad(8)$$

From (8) and (5) to (7), the differential equations are

$$\frac{dV_{LAB}}{dt} = \frac{1}{3C_f} i_{iAB} - \frac{1}{3C_f} (i_{LAB})$$

$$\frac{dV_{LBC}}{dt} = \frac{1}{3C_f} i_{iBC} - \frac{1}{3C_f} (i_{LBC})$$

$$\frac{dV_{LCA}}{dt} = \frac{1}{3C_f} i_{iCA} - \frac{1}{3C_f} (i_{LCA}) \qquad \dots (9)$$

By applying Kirchhoff's Voltage law on inverter output,

$$\frac{di_{iAB}}{dt} = -\frac{1}{L_f} V_{LAB} + \frac{1}{L_f} (V_{iAB})$$

$$\frac{di_{iBC}}{dt} = -\frac{1}{L_f} V_{LBC} + \frac{1}{L_f} (V_{iBC}) \qquad \dots (10)$$

$$\frac{di_{iCA}}{dt} = -\frac{1}{L_f} V_{LCA} + \frac{1}{L_f} (V_{iCA})$$

Then finally applying Kirchhoff's law to the load side,

$$V_{LAB} = L_{load} \frac{di_{LA}}{dt} + R_{load} i_{LA} - L_{load} \frac{di_{LB}}{dt} - R_{load} i_{LB}$$

$$V_{LBC} = L_{load} \frac{di_{LB}}{dt} + R_{load} i_{LB} - L_{load} \frac{di_{LC}}{dt} - R_{load} i_{LC}$$

$$V_{LCA} = L_{load} \frac{di_{LC}}{dt} + R_{load} i_{LC} - L_{load} \frac{di_{LA}}{dt} - R_{load} i_{LC}$$
....(11)

Equation (11) can also be re-written as,

$$\frac{di_{LAB}}{dt} = -\frac{R_{laod}}{L_{load}}i_{LAB} + \frac{1}{L_{load}}V_{LAB}$$

$$\frac{di_{LBC}}{dt} = -\frac{R_{laod}}{L_{load}}i_{LBC} + \frac{1}{L_{load}}V_{LBC} \qquad \dots (12)$$

$$\frac{di_{LCA}}{dt} = -\frac{R_{laod}}{L_{load}}i_{LCA} + \frac{1}{L_{load}}V_{LAB}$$

Then (9),(10) and (12) are written in matrix form by

$$\frac{dV_L}{dt} = \frac{1}{3C_f} I_i - \frac{1}{3C_f} I_L$$

$$\frac{dI_i}{dt} = -\frac{1}{L_f} V_L + \frac{1}{L_f} V_i \qquad \dots (13)$$

$$\frac{dI_L}{dt} = \frac{V_L}{L_{load}} - \frac{R_{load}}{L_{load}} I_L$$

Finally state space equation for LC filter circuit is given in (14).

$$\dot{X}(t) = AX(t) + Bu(t) \qquad \dots (14)$$

$$X = \begin{bmatrix} V_L \\ I_i \\ I_L \end{bmatrix}_{9\times 1}, A = \begin{bmatrix} 0_{3\times 3} & \frac{1}{3C_f} I_{3\times 3} & \frac{-1}{3C_f} I_{3\times 3} \\ \frac{-1}{L_f} I_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} \\ \frac{1}{L_{load}} I_{3\times 3} & 0_{3\times 3} & \frac{-R_{load}}{L_{load}} I_{3\times 3} \end{bmatrix}_{9\times 9}$$
$$B = \begin{bmatrix} 0_{3\times 3} \\ \frac{1}{L_f} I_{3\times 3} \\ 0_{3\times 3} \end{bmatrix}_{9\times 3}, u = [V_i]_{3\times 1} \qquad \dots (15)$$

Where
$$V_L = [V_{LAB} V_{LBC} V_{LCA}]^T$$
, $Ii = [i_{iAB}i_{iBC}i_{iCA}]^T$,
 $Vi = [V_{iAB}V_{iBC}V_{iCA}]^T$, $I_L = [i_{LAB}i_{LBC}i_{LCA}]^T$

4.0 SPWM TECHNIQUE AND HARMONIC ELIMINATION METHOD

4.1 SPWM Technique

The SPWM technique is very simple and very easy to implement. This method produces a sinusoidal waveform by filtering an output pulse waveform by varying width. The required output voltage is achieved by varying the amplitude and frequency of modulating voltage. The pulse width can be changed by changing the amplitude and frequency of reference or modulating voltage. In Figure 4 modulating wave is compared with high frequency triangular wave from. The high switching frequency leads to better output sinusoidal wave from. The switching state is changed when sine waveform is intersecting with high frequency triangular waveform [5-6].



In 3-phase VSI, the SPWM is achieved by three sinusoidal voltages (V_a , V_b , V_c) which are 120°out of phase with each other. These are compared with high frequency triangular waveform(V_T) and relative levels of the waveforms are used to control the switching of the devices in each phase leg of the inverter.



3-phase VSI having six switches (S_1-S_6) with each phase output connected to middle of the each inverter leg is shown in Figure 5. The output of the comparator forms the control signal for each leg of the inverter. In one lag, two switches makes a phase and these two switches open and close in a complementary fashion. The total voltage is V_{dc} , then the each pole voltage V_{ao} , V_{bo} , V_{co} of the inverter varies between- $V_{dc/2}$ and $+V_{dc/2}$. If the sine wave is greater than triangular wave, then upper switch is getting turned ON and lower switch is turned OFF. Based on switching states, positive or negative half DC link voltage is applied to each phase. Usually the switches are controlled in pairs (S_1, S_4) , (S_3, S_6) and (S_5, S_2) and the logic is shown in Table 1.

TABLE 1	
SWITCHING STATES	
S1 is ON when $V_a > V_T$	S4 is ON when $V_a < V_T$
S3 is ON when $V_b > V_T$	S6 is ON when $V_b < V_T$
S5 is ON when $V_c > V_T$	S2 is ON when $V_c < V_T$

4.2 HARMONIC ELIMINATION PRINCIPLE

Harmonic elimination problem is formulated by set of nonlinear transcendental equations and these equations must be solved in order to determine the switching angles so as to produce desired fundamental component which eliminates selected harmonics. The fundamental component is assigned a desired output value and the other selected orders of harmonics are requested to form the set of transcendental equations. By solving these equations switching angles are calculated to eliminate selected harmonics. Therefore, switching angles play an important role to produce the desired output by eliminating selected harmonics [7-12].

The PWM inverter output voltage contains harmonics and the output function f(t) can be expressed in terms of Fourier series as

$$f(t) = \sum_{n=1}^{\infty} (a_n sinn\alpha_n + b_n cosn\alpha_n) \qquad \dots (16)$$

If the output voltage wave is quarter wave symmetry of the output voltage, the even harmonics are absent and only odd harmonics are present. The amplitude of the n-th harmonic a_n is expressed only with first quadrant switching angles $\alpha_1, \alpha_2, \dots, \alpha_m$.

$$a_n = \left(\frac{4}{n\pi}\right) [1 + 2\sum_{k=1}^m (-1)^k \cos n\alpha_k] \qquad \dots (17)$$

and

$$0 < \alpha_1 < \alpha_2 < \cdots \dots \alpha_m < (\frac{\pi}{2})$$

In selected harmonic elimination, α_n is assigned the desired value for fundamental component and equated to zero for the harmonics to be eliminated

$$a_1 = \left(\frac{4}{\pi}\right) [1 + 2\sum_{k=1}^{m} (-1)^k \cos\alpha_k] = M \quad \dots (18)$$

$$a_5 = \left(\frac{4}{5\pi}\right) \left[1 + 2\sum_{k=1}^m (-1)^k \cos 5\alpha_k\right] = 0 \dots (19)$$

$$a_n = \left(\frac{4}{n\pi}\right) \left[1 + 2\sum_{k=1}^m (-1)^k \cos(\alpha_k)\right] = 0 \dots (20)$$

Nonlinear transcendental equations are thus formed and after solving these equations, α_1 through α_k are computed. Triplen harmonics are eliminated in three phase balanced system. It is evident that (m-1) harmonics can be eliminated with m no of switching angles [13-16].

The equation which is derived for Total Harmonic Distortion (THD) of the output voltage of an inverter is used in order to reduce the harmonics produced by the inverter. The THD is given by

$$a_n = \left(\frac{4}{n\pi}\right) [1 + 2\sum_{k=1}^m (-1)^k \cos \alpha_k] = 0 \dots (21)$$

where $n = 6i \pm 1$ and $i = (1,2,3 \dots)$

$$\%THD = \left[\frac{1}{(a_1)^2} \sum_{n=5}^{\infty} (-1)^k cosn\alpha_k\right) \qquad \dots (22)$$

5.0 SIMULATION RESULTS

The modeling and simulation has been done for SPWM technique using Matlab/Simulink. The system parameters for 3-phase VSI are given in Table 2.

TABLE 2	
3 PHASE INVERTER SYSTEM PARAMETERS	
DC-link Voltage V _{dc}	400 V
Fundamental Frequency f	50 Hz
Switching Frequency f _z	3 kHz
Modulation Index	M=0.8
Output Filter	$L_{f} = 800 \ \mu F$,
	$C_{\rm f} = 400 \ \mu F$
Load	$L_{load} = 2 mH$,
	$R_{load} = 5\Omega$

The Carrier wave (V_{tri}) and modulating wave (V_{sin}) are shown in Figure 6.

Inverter output line to line voltages are given in Figure 7. The voltages are called as V_{iAB} , V_{iBC} , V_{iCA} . Figure 8 shows inverter output currents i_{iA} , i_{iB} , i_{iC} .







Load line to line voltages and load phase currents i_{LA} , i_{LB} , i_{LC} are given in Figure 9 and Figure 10.





Inverter output line to line voltages (V_{iAB}), Inverter output currents (i_{iABC}), load line to line voltages (V_{LABC}) and load phase currents (i_{LABC}) are given in Figure 11.



Harmonic elimination problem is analyzed by solving set of nonlinear transcendental equations to determine the switching angles so as to produce desired fundamental component which eliminates selected harmonics. The switching angles which are required for the THD are calculated. The Figures 12-15 show variation of switching angles with respect to modulation index.









The proposed method has been simulated and the total harmonic distortion was calculated for output voltage and current of the inverter. The calculated THD is 3.2% and harmonic content for output voltage and current is shown in Figure16 and Figure 17.





6.0 CONCLUSION

With increasing effort in the direction of energy efficiency and power quality issues, grid connected solar PV systems are taking a good place. In this paper SPWM method is proposed for 3-phase grid connected VSI. The LC filter circuit is used in the proposed system. Complete mathematical model of this filter circuit is derived by using state space analysis and complete state space model is obtained and presented in this paper. The SPWM technique is implemented and simulated on 3 phases VSI using state space model of the LC filter circuit for grid connected solar PV system. Another important work presented in this paper is harmonic elimination. Harmonic elimination is done in VSI using Pulse Width Modulation (PWM) technique by solving the non-linear equations. Equations are used to determine switching angles of an inverter. The switching angle plays an important role to produce desired output by eliminating selected harmonics. Various simulation results are analyzed and presented on the inverter and load side of the proposed system in order to demonstrate the satisfactory performance of sine-PWM technique and harmonic elimination method for the grid connected solar PV system.

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