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Node Ordering Scheme of Large Scale Power Systems Using Sparse Matrix Techniques

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Power flow is the basic tool for power system analysis which reveals the system operation in a steady-state mode for evaluation of the power system planning and operations. The accuracy, simulation time, computer storage size and convergence of any model used depend largely on the size of the bus admittance matrix of the system under study. This paper, therefore, presents the study of the bus admittance matrix of the different systems with sparse techniques. And also analyze the number of zeros and non-zeros element in the different systems with minimum ordering schemes. The proposed method is validated using a 5-bus, 30-bus, 118-bus and 300-bus systems. The results are presented in graphical form and discussed. The sparse Matrix techniques show that as the system is increasing in size, the percentage of stored bus admittance elements decreases and changing the order of the nodes gives more impact on the size of the system. Thus, an appreciable reduction in the computer memory required to store the bus admittance matrix and in turn reduces the overall simulation time.

Keywords: *Sparse Matrix Techniques, Non-zero element and LU Factorization.*

1.0 INTRODUCTION

Power flow analysis is an important tool during planning stages of new power system or addition to existing ones like adding new generator sites, meeting increase load demand and locating new transmission sites [1]. In power flow analysis, some power system parameters are important in determining the system performance. These parameters include voltage magnitude and angle at every bus of the system, real and reactive power injections at all the buses and power flows through interconnecting power channels, reactive power flows along the transmission lines, real and reactive power losses along the transmission lines and total losses [2]. In the study of electric power systems, several different researches have been carried out by different researchers using different methods. The most commonly used methods for power flow study are Gauss-Seidel, Newton-Raphson and fast decoupled.

Among all these methods, Newton-Raphson is found to be the most widely used model in power systems applications [3]. However, these conventional power flow algorithms are not efficient due to the challenges they encountered, which in most cases, results to divergence of the algorithms. These challenges include singularity of the Jacobian matrix most especially in Newton-Raphson method, large memory capacity for storing the elements of the Jacobian and the bus admittance matrix, convergence error or problem, high or large simulation time etc.

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To combat the problems inherent in divergence algorithm, various modifications have been used by different researchers in the past [5]. This paper provides an idea of sparse Matrix techniques on IEEE bench-mark systems to gives excellent method for computation of the power systems. All practical power systems have majority or larger percentage of their buses not connected through transmission lines. This characteristic is explored in this paper such that only non-zero elements are stored and thereby resulting to savings in computer memory (the CPU time per iteration is made to be relatively small). The savings in computer memory is very important when dealing with large practical power systems to reduce the computation time [4] by reducing large memory required for the storage of the elements of bus admittance matrix.

The nodal admittance matrix of the power network is typically very sparse. However, the equivalent model after network reduction is usually denser (less sparse) because the connections between retained buses have to be preserved after reduction. All nodes that were connected to the deleted node (boundary nodes) are mutually connected, whether they were originally directly connected or not. In order to get a reduced model that is easier to analyze than the original one it is necessary to preserve its sparsity. High number of new branches can decrease the effect of nodes reduction on the sparsity of the model and the impedance of new branches can sometimes be very high leading to numerical problems. The sparse Matrix techniques show that as the system is increasing in size, the percentage of stored bus admittance elements decreases and changing the order of the nodes gives more impact on the size of the system.

2.0 SPARSITY SOLUTION TECHNIQUES

In large power systems, each bus is connected to only a small number of other buses. Therefore, bus admittance matrix of a large power system is very sparse. This means that the bus admittance matrix will contain larger percentage of zeros as compared to the non-zero elements. This characteristic feature shows a considerable reduction in the storage handling of the computer which indicates a substantial improvement compared to the work reported in literature. This sparsity feature of Ybusmatrix also extends to Jacobian matrix. According to reference [6], "Sparsity can be simply defined to indicate the absence of certain problem interconnections". Mathematically, the sparsity of an n×n matrix is given by reference [5] as

$$\text{Sparsity} = \left[\frac{\text{Totalno. of Zero elements}}{\text{n}^2} \times 100\%\right] \quad ....(1)$$

In a large power system such as the ones considered in this work, sparsity may be as high as 97%. Though Y_{bus} is sparse, Z_{bus} is full. This sparsity is employed in this work to ensure that only the non-zero elements are stored and the full characteristic of the original matrix is not lost.

2.1 Triangular Factorization

To solve the generalized jacobian matrix equation represented here as

$$[\Delta S] = [J][\Delta E] \quad ....(2)$$

For increments in voltage, the direct method is to find the inverse of [*J*] and solve for [Δ*E*] from

$$[\Delta E] = [J]^{-1}[\Delta S] \quad ....(3)$$

In power systems [*J*] is usually sparse but $[J]^{-1}$ is a full matrix. The method of triangular factorization solves for the vector [Δ*E*] by eliminating [*J*] to an upper triangular matrix with a leading diagonal and then back-substituting for [Δ*E*] i.e., eliminate[1] to

$$[\Delta S'] = [U][\Delta E] \quad ....(4)$$

and back-substitute

$$[U]^{-1}[\Delta S'] = [\Delta E] \quad ....(5)$$

Where

$[U]$ = Upper Triangular Factorization

$[J]$ = Jacobian matrix

$[\Delta E]$ = Nodal Voltages

The triangulation of the Jacobian is best done by rows. Those rows below the one being operated on need not be entered until required. This means that the maximum storage is that of the resultant upper triangle and diagonal. The lower triangle can then be used to record operations. The number of multiplications & additions to triangulate a full matrix is $1/3N^3$, compared to N^3 to find the inverse with sparsity programming the number of operations varies as a factor of N. If rows are normalized N further operations are saved.

2.2 Node Ordering Schemes

Node ordering schemes are important in minimizing the number of multiplications and divisions required for both L & U triangularization and forward/backward substitution. A good ordering will result in the addition of few fills to the triangular factors during the LU factorization process. A fill is a non-zero element in the L or U matrix that was zero in the original A matrix.

If A is a full matrix, $\propto = \left[\frac{n^3 - n}{3}\right]$ multiplications & divisions are required for the LU Factorization process and $\beta = n^2$ multiplications and divisions are required for the forward / backward substitution process. The number of multiplications and divisions required can be substantially reduces in sparse matrix solutions if a proper node ordering is used.

3.0 IMPLEMENTATION

The triangular factorization and Minimum order scheme is implemented on the IEEE standard bus system. The below Figure 1 to Figure 3 is for 5-bus system and Figure 4 to Figure 6 is for 300-bus sytem. The details of the plots as follows. The Figure 1 and Figure 4 is the jacobian matrix of the 5 and 300 bus systems. A Figure 2 and Figure 5 are the LU factorization yields matrices of the 5 and 300 bus system. Figure 2 and Figure 5 notice that two sub diagonals have created a large number of fills and extend between them and the main diagonal. This factorization gives more impact on memories and computation time.

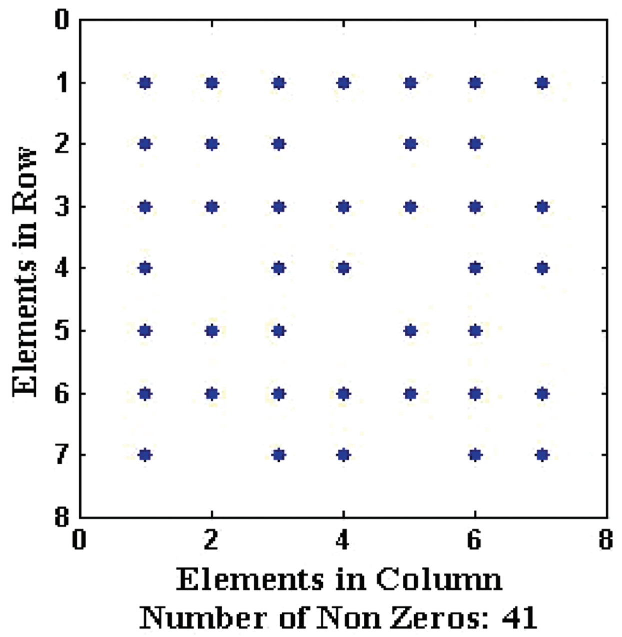

FIG. 1 JACOBIAN MATRIX OF 5-BUS SYSTEM

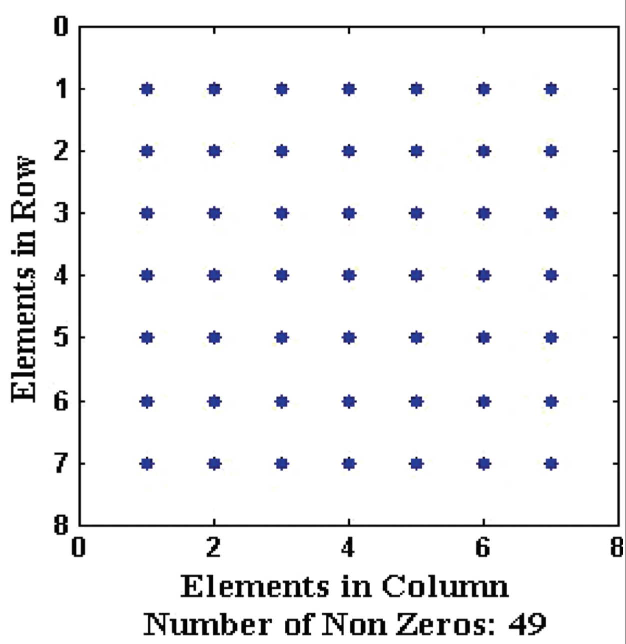

FIG. 2 LU FACTORIZATION MATRIX OF 5-BUS SYSTEM

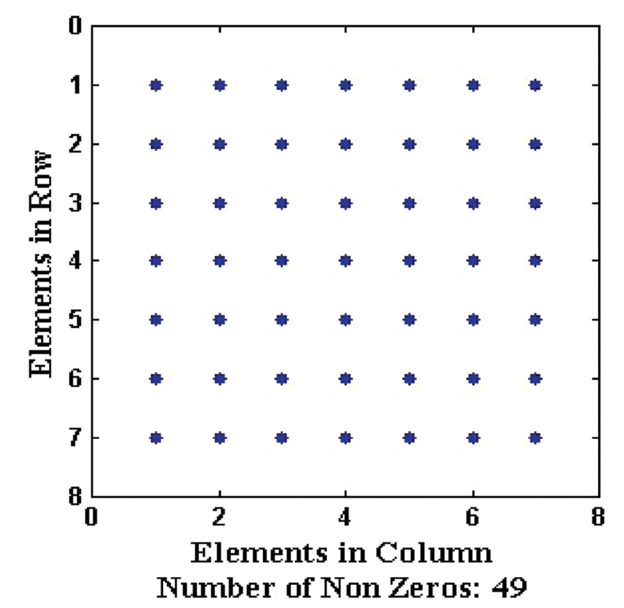

FIG. 3 REORDERED LU FACTORS OF 5-BUS SYSTEM

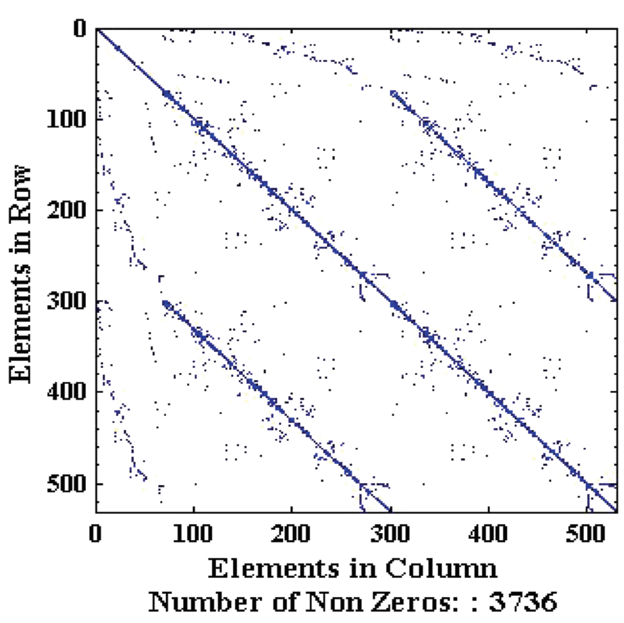

FIG. 4 JACOBIAN MATRIX OF 300-BUS SYSTEM

The Sparse column minimum degree permutation schemes are used to minimize the number of fills in the matrix. The Figure 3 and Figure 6 shows that having the 49 & 9889 non -zero element in the 5-bus and 300-bus system. This Minimum ordering scheme reduces 70% of the non-zero element in the system.

4.0 RESULTS AND DISCUSSION

The results of the simulation, using the proposed techniques, validated using the standard bus systems are presented in Table 1 and 2. In the Table 1, Shows that number of non-zero element present in the system is compared with the total element in the full matrix. In the Table 1 shows that, 5-bus system has 76% ratio compare with the 300-bus system has only 1.24% variation.

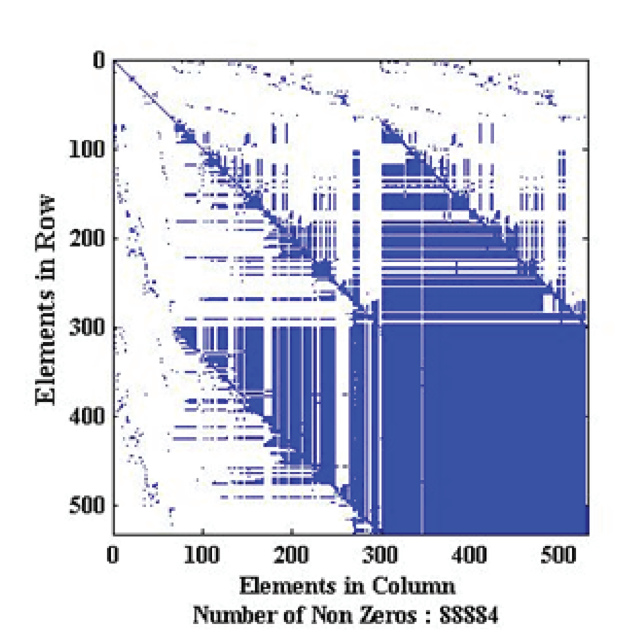

FIG. 5 LU FACTORIZATION MATRIX OF 300-BUS SYSTEM

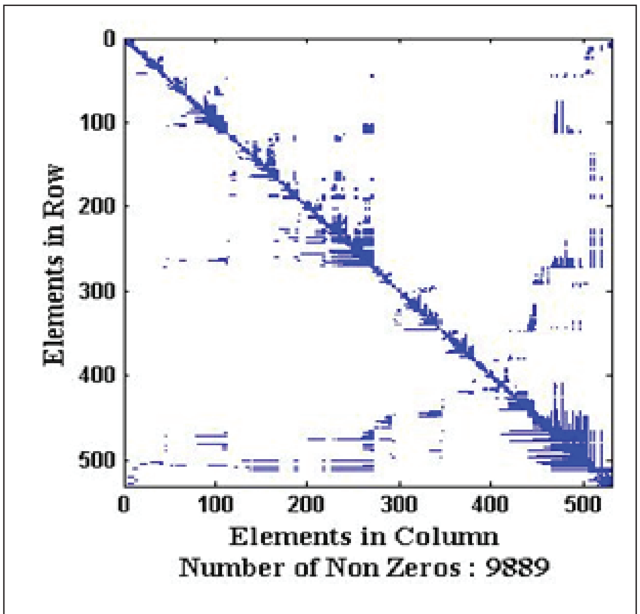

FIG. 6 REORDERED LU FACTORS OF 300-BUS SYSTEM

TABLE 1			
PERCENTAGE RATIO OF NON-ZERO ELEMENT IN THE MATRIX			
No. of Bus in the system (N)	**No. of element in Full Matrix (N^2)**	**No. of non- Zero element**	**% Ratio (Sparsity)**
5	25	19	76.00%
30	900	112	12.44%
118	13924	476	3.42%
300	90000	1118	1.24%

In the Table 1 summarized that memory of the system also depends upon the size of the system. Normally, the single element takes 8 bytes in the memory of the computer. In the above Table 2 shows that, the 5-bus system has 93.18% ratio compare with the 300-bus system has only 11.82% using minimum ordering scheme.

TABLE 2			
PERCENTAGE RATIO OF MEMORY SPACE OF THE LU FACTORS & MINIMUM ORDERING			
No. of Bus in the system	**LU Factorization Memory space**	**Minimum ordering scheme-memory**	**% Ratio**
5	704 Bytes	656 Bytes	93.1%
30	20352 Bytes	6924 Bytes	34.0%
118	158084 Bytes	23300 Bytes	14.7%
300	1082424 Bytes	127944 Bytes	11.8%

From the Table 1 & 2 summarized that the size of the system and the reordering of the nodes gives more impact on the computation of the simulation.

5.0 CONCLUSION

Node Ordering Scheme shows that the number of the zero elements in the system is depends upon the size of the system. If the system size is increasing, the no. of zero elements also increases. Same as the proper ordering also give more impact on the size of the system. By using the ordering scheme, we can minimize the number of non-zero elements and decrease the computation time and memory space of the system. So it is concluded that the non-zero element in the system is decreasing when the size of the system is increasing.

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