

Mathematical modeling and simulation of an observer based state feedback controller of a STATCOM to improve the voltage stability of power system

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Power demand in India is increasing significantly day by day due to the growing population, dynamic economic growth and modernization. To meet such large and growing power needs our power system is facing power quality problems inform of low power factor, poor voltage profile, voltage fluctuations, voltage sag/swell etc. So it is indeed very essential to improve the power transfer capability of existing networks. Installation of FACTS device in existing transmission network is an alternative way to strengthen power transmission capability. A static synchronous compensator (STATCOM) is one of the FACTS devices which is used to control the reactive power and it has also the ability to regulate the grid voltage. Different control techniques are in use to control the amount of reactive power to be injected or supplied by the STATCOM. In this paper, an OBSERVER based state feedback controller is proposed for STATCOM to improve the voltage stability. Mathematical modeling for the controller is also illustrated here. Simulation of observer based state feedback control of STATCOM is performed in MATLAB/SIMULINK. The simulation result depicts the satisfactory performance of STATCOM with proposed controller during three phases to ground fault condition.

Keywords: *FACTS, observer, STATCOM, state feedback controller, voltage stability.*

1.0 INTRODUCTION

FACTS are power electronic based static devices, which are used to increase the power transfer capacity and enhance controllability in ac transmission system [7]. FACTS devices can be connected to a transmission network in different ways, like as in series, shunt, or a combination of series and shunt. Series FACTS devices increase stability and Shunt FACTS devices provide reactive power compensation.

The STATCOM is a second generation FACTS device. It is classified as a shunt compensator. A STATCOM consists of a three phase inverter and a D.C capacitor which provides the D.C voltage for the inverter. From the dc side capacitor, a three

phase voltage is generated by the inverter. This is synchronized with the ac supply. STATCOM can absorb as well as generate reactive power and this is achieved by controlling the firing angle of the inverter. Different controllers are in use with STATCOM to control the firing angle of the inverter.

In this paper an observer based state feedback controller is proposed with STATCOM to generate control signal. The mathematical explanation of STATCOM based on state space theory and the proposed controller model is also presented here. The voltage equations are transformed into d-q reference frame. The controller is tested on a transmission system under MATLAB/SIMULINK Environment.

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2.0 DYNAMIC MODEL OF STATCOM

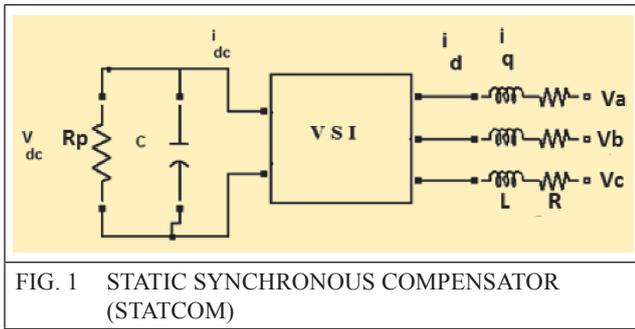


FIG. 1 STATIC SYNCHRONOUS COMPENSATOR (STATCOM)

Figure 1 shows the STATCOM based on VSI topology. The dynamic equation of Figure 1 can be written as:

$$\frac{di_a}{dt} = -\frac{R}{L}i_a + \frac{1}{L}V_a - \frac{1}{L}e_a \quad \dots(1)$$

$$\frac{di_b}{dt} = -\frac{R}{L}i_b + \frac{1}{L}V_b - \frac{1}{L}e_b \quad \dots(2)$$

$$\frac{di_c}{dt} = -\frac{R}{L}i_c + \frac{1}{L}V_c - \frac{1}{L}e_c \quad \dots(3)$$

$$\frac{dV_{dc}}{dt} = -\frac{V_{dc}}{R_p C} + \frac{1}{C}i_{dc} \quad \dots(4)$$

Now equations (1), (2) and (3) can be transformed to synchronously rotating reference frame as:

$$\frac{di_d}{dt} = -\frac{R}{L}i_d + \frac{1}{L}V_d - \frac{1}{L}e_d + \omega i_q \quad \dots(5)$$

$$\frac{di_q}{dt} = -\frac{R}{L}i_q + \frac{1}{L}V_q - \frac{1}{L}e_q + \omega i_d \quad \dots(6)$$

The magnitude of converter voltage and dc voltage across capacitor is directly proportional to each other.

$$e_d = mV_{dc} \cos \theta \quad \dots(a)$$

$$e_q = -mV_{dc} \sin \theta \quad \dots(b)$$

Where, m is the co-efficient related to the type of the converter.

Using equation (a) and (b) the equation (5) and (6) can be written as:

$$\frac{di_d}{dt} = -\frac{R}{L}i_d + \frac{1}{L}V_d - \frac{1}{L}mV_{dc} \cos \theta + \omega i_q \quad \dots(7)$$

$$\frac{di_q}{dt} = -\frac{R}{L}i_q + \frac{1}{L}V_q + \frac{1}{L}mV_{dc} \sin \theta + \omega i_d \quad \dots(8)$$

Now Power,

$$\begin{aligned} P &= V_{dc}i_{dc} = \frac{3}{2}(e_d i_d + e_q i_q) \\ &= \frac{3}{2}(i_d mV_{dc} \cos \theta + i_q mV_{dc} \sin \theta) \end{aligned}$$

[From (a) and (b)]

Now,

$$i_{dc} = \frac{3}{2} m(i_d \cos \theta + i_q \sin \theta) \quad \dots(9)$$

Now using equation (9) the equation (4) can be written as,

$$\begin{aligned} \frac{dV_{dc}}{dt} &= -\frac{V_{dc}}{R_p C} + \frac{3}{2C} m i_d \cos \theta \\ &\quad - \frac{3}{2C} m i_q \sin \theta \end{aligned} \quad \dots(10)$$

Now the equation (7), (8) and (10) can be written as matrix form like:

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \\ V_{dc} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega & -\frac{m}{L} \cos\delta \\ -\omega & -\frac{R}{L} & \frac{m}{L} \sin\delta \\ \frac{3m}{2C} \cos\delta & -\frac{3m}{2C} \sin\delta & -\frac{1}{R_p C} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ V_{dc} \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix} \dots(11)$$

Now the state space equation as:

$$\dot{X} = Ax + Bu \dots(a)$$

$$y = Cx + Du \dots(b)$$

Comparing equation (b) with equation (11) we get:

$$A = \begin{bmatrix} -\frac{R}{L} & \omega & -\frac{m}{L} \cos\delta \\ -\omega & -\frac{R}{L} & \frac{m}{L} \sin\delta \\ \frac{3m}{2C} \cos\delta & -\frac{3m}{2C} \sin\delta & -\frac{1}{R_p C} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \\ 0 & 0 \end{bmatrix} \dots(12)$$

The characteristic equation is $|(SI - A)|=0$.

By solving this, the characteristic equation of STATCOM is:

$$\begin{aligned} |(SI - A)| &= S^3 + S^2 \left(\frac{2}{T_1} + \frac{1}{T_2} \right) \\ &+ S \left(\frac{2}{T_1 T_2} + \frac{1}{T_1^2} + \frac{3m^2}{2LC} + \omega^2 \right) \\ &+ \left(\frac{3m^2}{2LCT_1} + \frac{1}{T_1^2 T_2} + \frac{\omega^2}{T_2} \right) \\ &= 0 \dots(13) \end{aligned}$$

Where, $T_1 = \frac{1}{R}$ and $T_2 = R_p C$

3.0 CONTROLLER DESIGN

3.1 State Feedback Controller

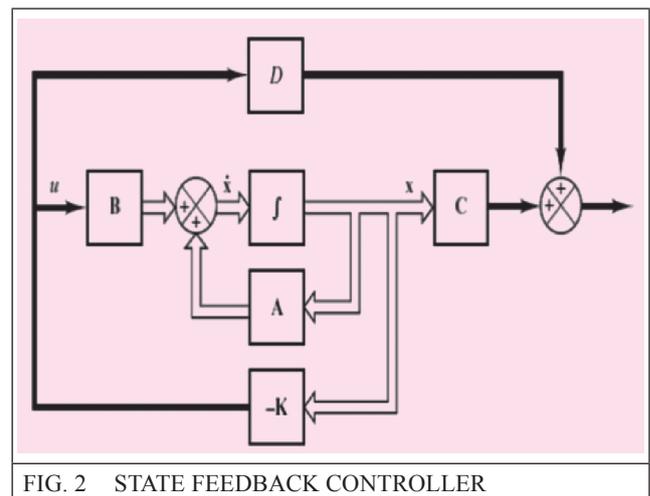


FIG. 2 STATE FEEDBACK CONTROLLER

State feedback is also known as pole placement method which is used in feedback control system to place the closed-loop poles of a system in pre-determined locations in the s-plane. The system should be controllable in order to implement this pole placement method. Figure 2 shows the block diagram of state feedback controller, where k is feedback gain matrix.

$$K = [K_1 \ K_2 \ K_3]$$

The roots of feedback system are given by the characteristic equation, $|(SI - (A - BK))|$.

By putting the values of A and B matrix, the characteristic equation is:

$$\begin{aligned}
 &S^3 + S^2 \left(2R + \frac{1}{R_p C} + \frac{k_1}{L} + \frac{k_2}{L} \right) \\
 &+ S \left(\frac{2Rk_2}{L} + \frac{k_2}{LR_p C} + \frac{2R}{R_p C} + R^2 + \frac{3m^2}{2LC} \right. \\
 &+ \left. \omega^2 + \frac{2k_1 R}{L} + \frac{k_1}{LR_p C} + \frac{k_1}{L} \right) \\
 &+ \left(\frac{2k_1 R}{LR_p C} + \frac{k_1 R^2}{L} + \frac{3m^2 k_1}{2L^2 C} + \frac{k_1 \omega^2}{L} + \frac{2Rk_1}{L} \right. \\
 &- \left. \frac{3m^2 R}{2LC} - \frac{R^2}{R_p C} - \frac{\omega^2}{R_p C} + \frac{3m^2 k_2}{2L^2 C} + \frac{k_2 R^2}{LR_p C} \right. \\
 &+ \left. \frac{\omega^2 k_2}{LR_p C} \right) \\
 &= 0 \qquad \dots(14)
 \end{aligned}$$

Let, the desired characteristics equation is,

$$S^3 + d_1 S^2 + d_2 S + d_3 = 0 \qquad \dots(15)$$

Now by comparing (14) and (15), the feedback gains are,

$$\begin{aligned}
 k_1 = L \left\{ d_2 - d_1 \left(2R - \frac{1}{R_p C} \right) \right. \\
 + \left(3R^2 + \frac{4R}{R_p C} + \frac{1}{R_p^2 C^2} \right. \\
 \left. \left. - \frac{3m^2}{2LC} - \omega^2 \right) \right\}
 \end{aligned}$$

$$\begin{aligned}
 k_2 = L \left\{ \left(d_1 - 2R - \frac{1}{R_p C} \right) - d_2 \right. \\
 + d_1 \left(2R - \frac{1}{R_p C} \right) \\
 - \left(3R^2 + \frac{4R}{R_p C} + \frac{1}{R_p^2 C^2} \right. \\
 \left. \left. - \frac{3m^2}{2LC} - \omega^2 \right) \right\}
 \end{aligned}$$

3.2 Observer based state feedback controller

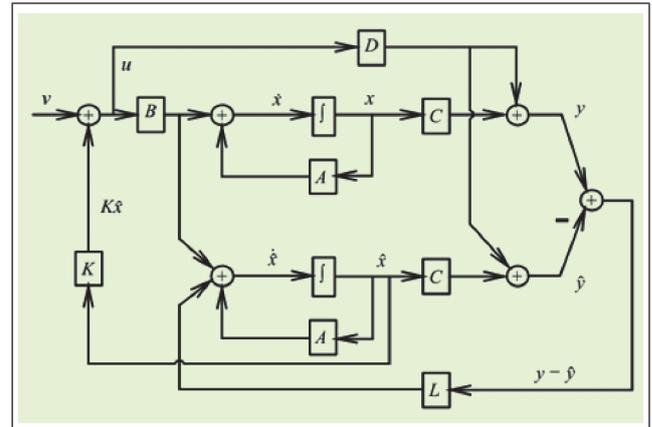


FIG. 3 OBSERVER BASED STATE FEEDBACK CONTROLLER

An observer is called an estimator which is used to calculate the internal state variables that are not accessible from the system. Figure 3 shows the block diagram of state feedback controller with observer where L is observer gain matrix.

$$L = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$$

The roots of the observer are given by the characteristic equation, $|SI - (A_x - L_x C)|$

Here $C = [0 \ 0 \ 1]$ and A_x and L_x is the canonical form of the characteristic polynomials.

Now by simplifying this, the characteristic equation is:

$$\begin{aligned}
 &S^3 + S^2 \left(2R + \frac{1}{R_p C} + l_1 \right) \\
 &+ S \left(\frac{2R}{R_p C} + R^2 + \frac{3m^2}{2LC} + \omega \right. \\
 &+ \left. l_2 \right) \\
 &+ \left(\frac{3m^2 R}{2LC} + \frac{R^2}{R_p C} + \frac{\omega^2}{R_p C} \right. \\
 &+ \left. l_3 \right) = 0 \qquad \dots(16)
 \end{aligned}$$

Now, let the desired characteristic equation:

$$S^3 + a_1S^2 + a_2S + a_3 = 0 \quad \dots(17)$$

Now by comparing (16) and (17), the observer gains are,

$$l_1 = a_1 - (2R + \frac{1}{R_pC})$$

$$l_2 = a_2 - (\frac{2R}{R_pC} + R^2 + \frac{3m^2}{2LC} + \omega)$$

$$l_3 = a_3 - (\frac{3m^2R}{2LC} + \frac{R^2}{R_pC} + \frac{\omega^2}{R_pC})$$

Based on these mathematical calculations we have designed a generalized model of the proposed controller of STATCOM and this is shown in Figure 4.

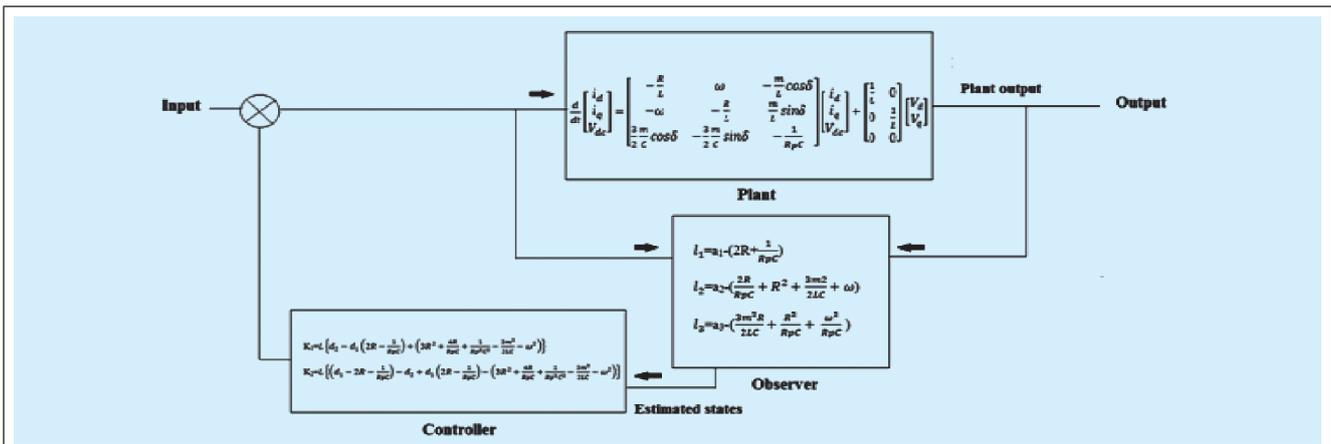


FIG. 4 GENERALIZED MODEL OF PROPOSED CONTROLLER

4.0 SIMULATION, RESULTS AND DISCUSSIONS

In order to verify the proposed approach, a MATLAB model for state observer is built to test the system. The parameters for STATCOM shown in Figure 1 are as follows:

$R_p = 0.5$ ohm, $C = 1e-6$ F, $V_{dc} = 400$ V, $L = 1.77e-3$ H, $R = 0.05$ ohm and $V_s = 230$ V.

In order to apply observer based state feedback controller, the system must be completely observable and controllable. So first of all we will check the controllability and observability of the plant (e.g. STATCOM).

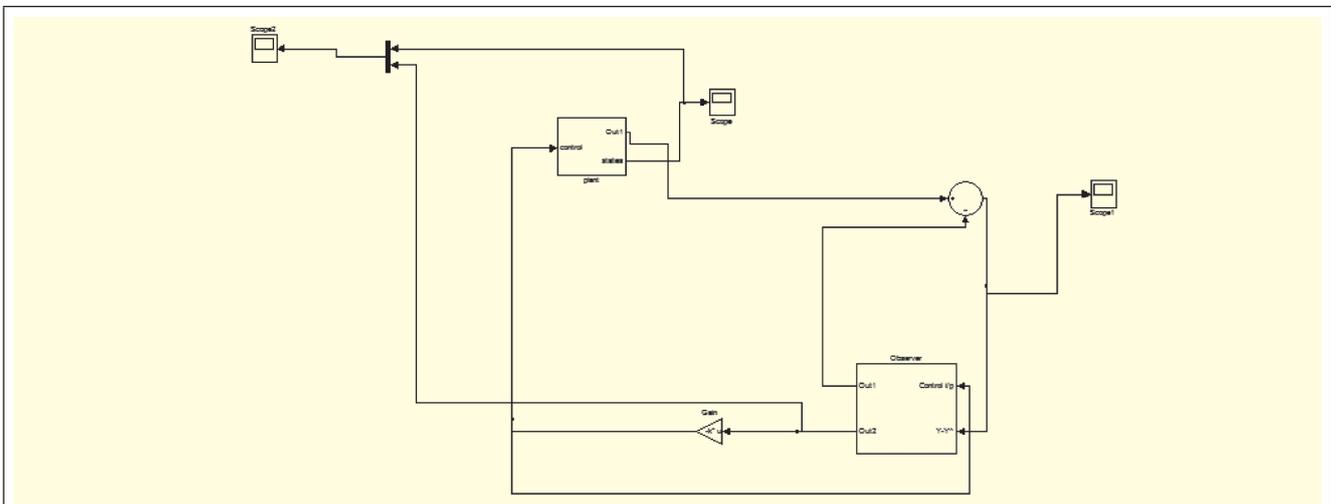


FIG. 5 SIMULATIONDIAGRAM OF PROPOSED CONTROLLER

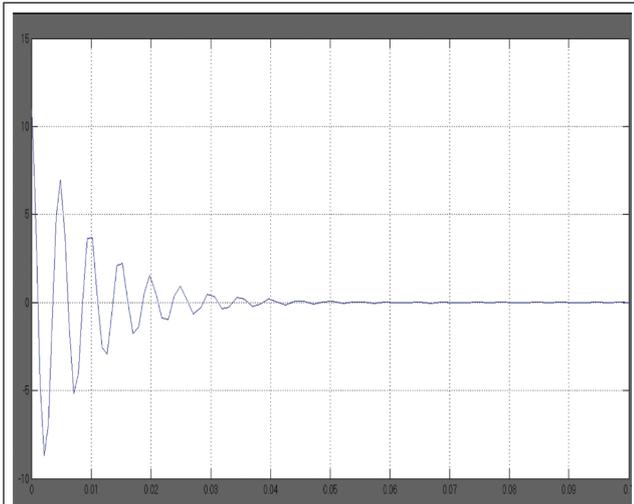


FIG. 6 GRAPH FOR ERROR SIGNAL

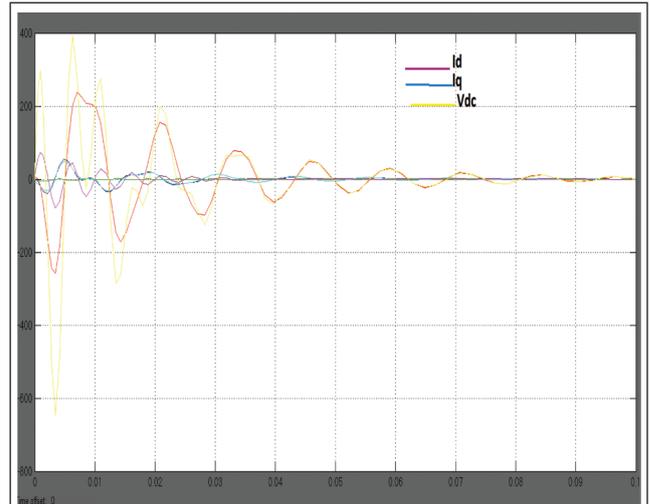


FIG. 8 GRAPH FOR OBSERVABILITY TEST

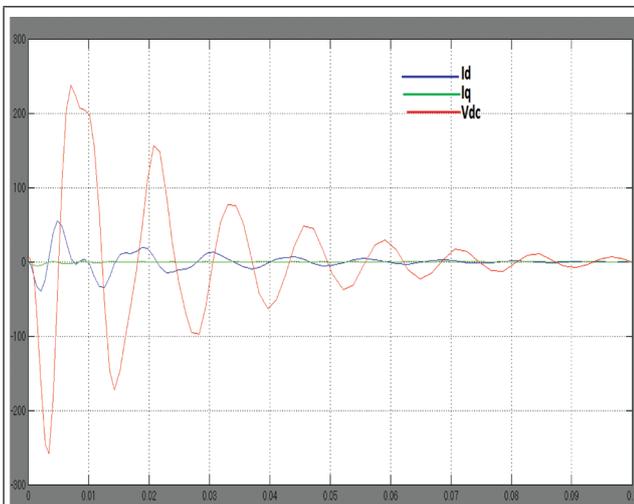


FIG. 7 GRAPH FOR CONTROLLABILITY TEST

Figure 5 is the simulation diagram of a state feedback controller with observer. Here, plant is STATCOM. The simulation result shows the controllability and observability of the system.

The simulation graph 6 is for the error signal means difference between two states. From the Figure 6 it is clear that the error is reduced to zero. That means observer based state feedback controller can minimize the error. Now Figure 7 is the graph for the state feedback controller and from this, it shows that the states of the STATCOM (I_d , I_q and V_{dc}) starting from any initial condition reduces to zero, which means

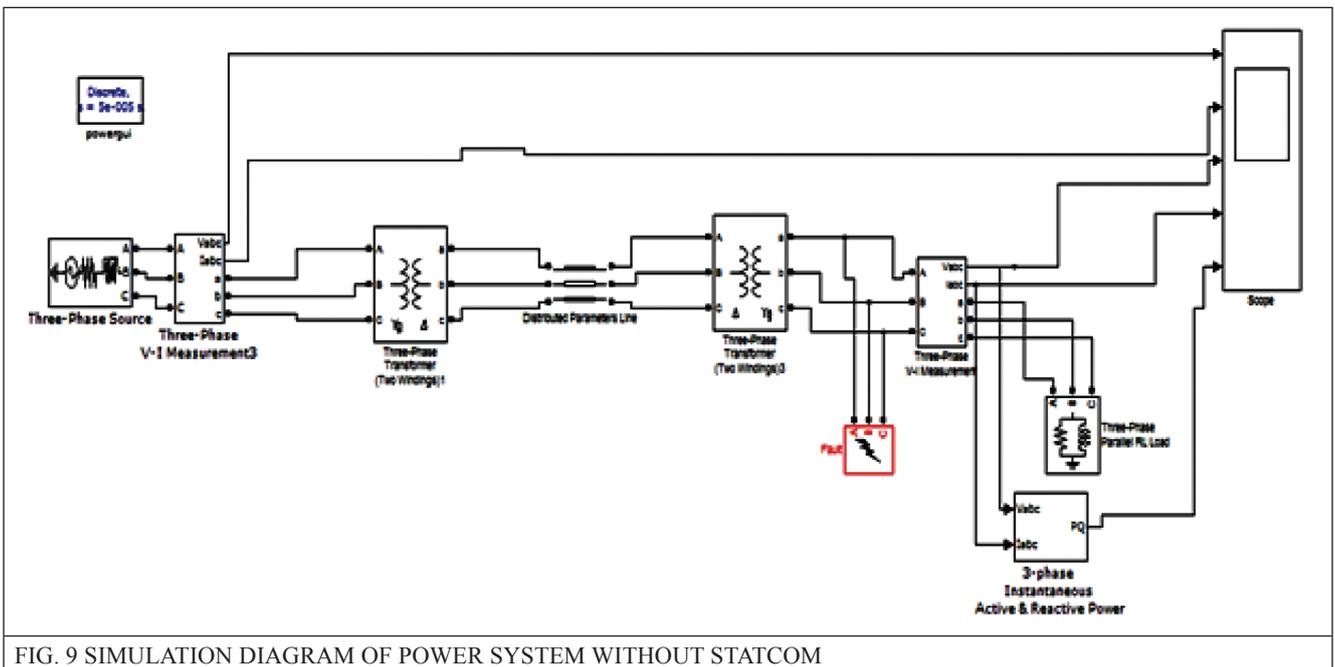


FIG. 9 SIMULATION DIAGRAM OF POWER SYSTEM WITHOUT STATCOM

system is fully controllable. Now Figure 8 is for the observer and it will give an idea, whether this observer is working or not. This graph displays the states of the mentioned plant and proves that, the states of observer is converged and reduces to zero, which means the observer is working properly and the system is completely observable.

Now in order to verify the proposed controller, a simplest power system is analyzed with and without STATCOM by the utilization of SimPower System toolbox presented in the MATLAB/SIMULINK software environment.

Figure 9 is the simulation diagram of a simple power system without STATCOM. The test system comprises of 11kV/33kV, 50 Hz distribution network with three phase parallel load. A three phase to ground fault is applied at the receiving end for 100 ms. From Figure 10 it is observed that, as the fault is in receiving end, so there is no change in the voltage and current of sending end. In Figure 11, it is clear that during the fault, the receiving end voltage reduced to 0 V from 1000 V and also current is reduced to 2A from 5A and active and reactive power [blue -> active power and green -> reactive power] also drops down to zero during that particular fault duration of 100 ms.

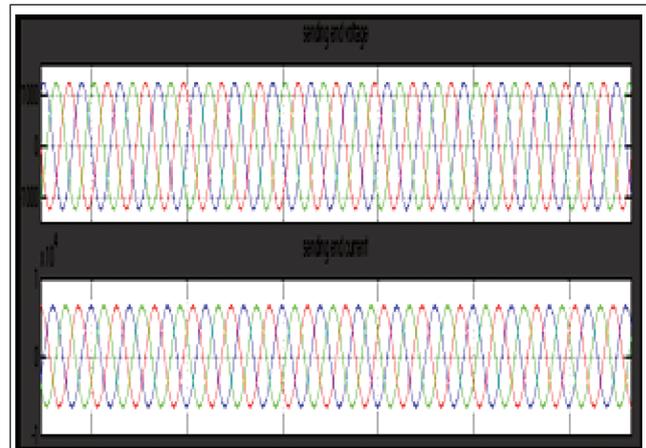


FIG. 10 SENDING END VOLTAGE AND SENDING END CURRENT WITHOUT STATCOM

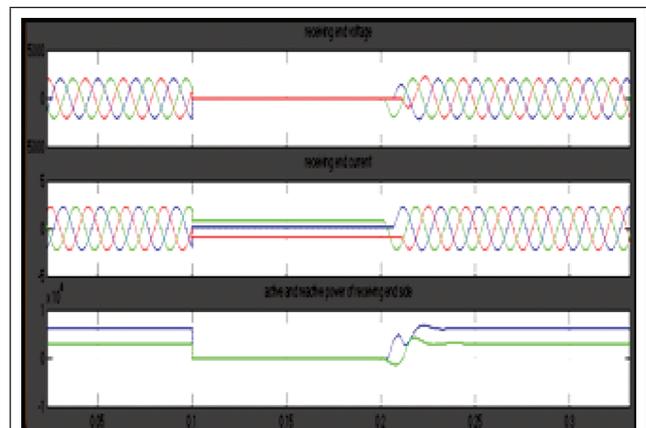


FIG. 11 RECEIVING END VOLTAGE, CURRENT, ACTIVE AND REACTIVE POWER WITHOUT STATCOM

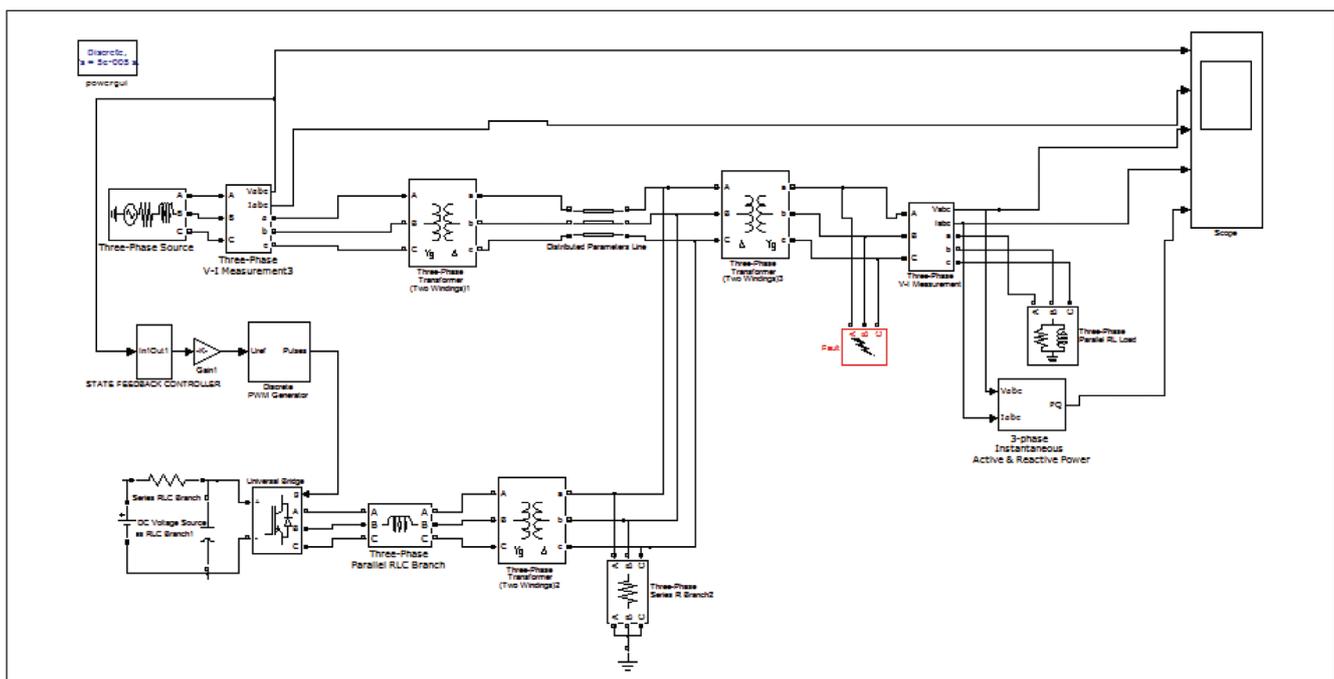


FIG. 12 SIMULATION DIAGRAM OF A SIMPLE POWER SYSTEM WITH OBSERVER BASED STATE FEEDBACK CONTROLLER BASED STATCOM

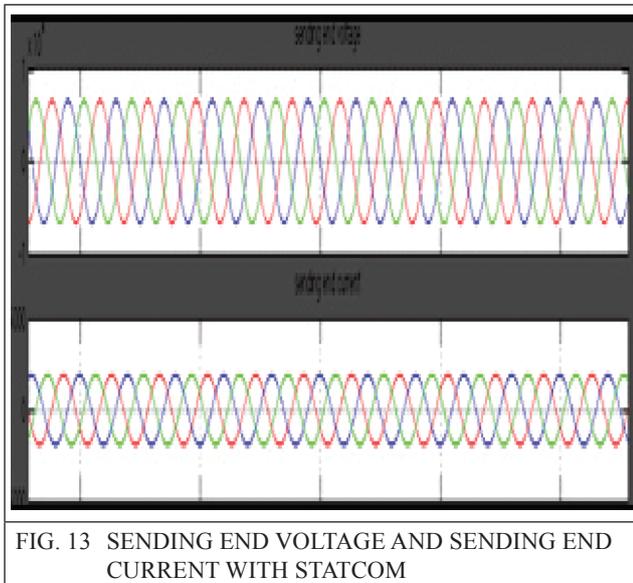


FIG. 13 SENDING END VOLTAGE AND SENDING END CURRENT WITH STATCOM

Figure 12 is the simulation diagram of a simplified power system network with a STATCOM connected in parallel with the transmission line. Here control signal is generated by the observer based state feedback controller. In this, 'Test System' a three phases to ground fault is also applied in the receiving end for 100 ms. Figure 13 shows the sending end voltage and sending.

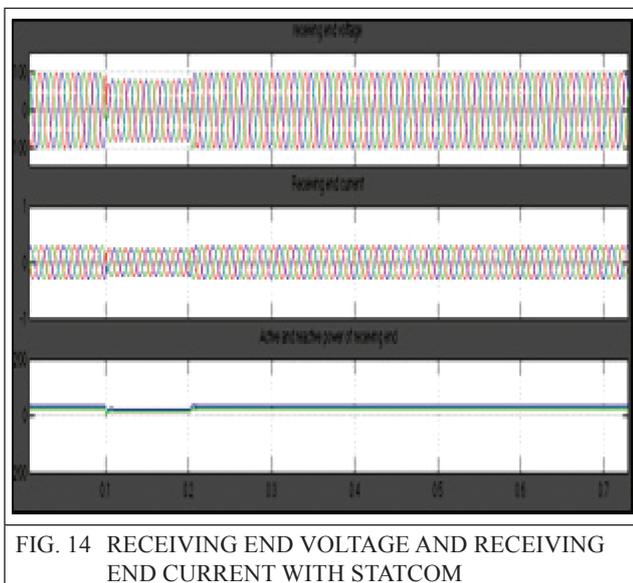


FIG. 14 RECEIVING END VOLTAGE AND RECEIVING END CURRENT WITH STATCOM

end current which is not effected due to the fault. In Figure 14, as STATCOM is connected to the system, the fault is cleared and observer based state feedback controller provides the reactive

support to the system, as a result voltage and current is also improved and reaches nearly to its rated value.

5.0 CONCLUSION

In this paper, simulation of STATCOM is performed using MATLAB/SIMULINK™. An observer based state feedback controller is proposed and implemented with STATCOM to get the satisfactory result. The dynamic model of STATCOM and the equations of the controller are also presented here. From the simulation result, it has been observed that the proposed controller is efficient and it can provide reactive support to the system when there is any unbalance condition in the system and thus, it can improve the voltage stability.

REFERENCES

- [1] S Pradeepa, K Uma Rao, Ravishankar Deekshit and Shantha M S, State-feedback Control of a Voltage Source Inverter-based STATCOM, IEEE International Conference on Power, Energy and Control, pp. 120-123, 2013.
- [2] M Homayounzade, M Keshmiri, M Danesh, An Observer-Based State Feedback Controller Design for Robot Manipulators Considering Actuators' Dynamic, IEEE 2010 15th International Conference on Methods and Models in Automation and Robotics (MMAR), pp. 240-248, 2010.
- [3] Ajami A., Younesi M., Modeling and state feedback controller for current source inverter based STATCOM, IEEE International Conference on Control, Automation and System, pp. 2418-2423, 2008.
- [4] Peyman Jafarian, Mohamad Tavakoli Bina State-Feedback Current Control of VSI-Based D-STATCOM for Load Compensation, IEEE International Conference on Environment and Electrical Engineering (EEEIC), pp. 214-217, 2010.

- [5] Nitus Voraphonpiput, Somchai Chatratana, STATCOM Analysis and Controller Design for Power System Voltage Regulation, IEEE Transmission and Distribution Conference & Exhibition, pp. 1-6, 2005.
- [6] Gang Yao, LiXue Tao, LiDan Zhou, Chen Chen, State-feedback Control of a Current Source Inverter-based STATCOM, ELEKTRONIKA IR ELEKTRO-TECHNIKA, pp. 98-102, 2010.
- [7] P S Ponmurugavel, S Mohamed Ghouse, Design and Modelling of Fuzzy and Model Predictive Controllers for STATCOM to enhance Transient Stability of Power, International Journal of Engineering and Technology (IJET), Vol 5, pp. 2609-2619, 2013.
- [8] Smriti Dey, Comparison of DVR and D-STATCOM for Voltage Quality Improvement, International Journal of Emerging Technology and Advanced Engineering, Vol 4, pp. 187-193, 2014.

