The Journal of CPRI,
Vol. 7, No. 1, March 2011 pp. 101–114

Load Flow Study of a Radial Distribution Network

Tejas Vyas* and Ranjit Roy**

The paper presents the simple method for finding the load flow solution of a radial distribution network using the Kirchoff's current and Kirchoff's voltage law. The method is tested on four test cases: 16-node, 33-node, 69-node and 117-node network. Comparing to the existing methods, the memory space requirement is very small and time required for the CPU execution is also lower. Result shows that the proposed method is very efficient and competitive with the existing methods.

1.0 INTRODUCTION

Loss minimization in power distribution system is one of the biggest challenges before power engineers. It has estimated that distribution systems cause a loss of about 5–13 % of the total power generated in developing countries. Therefore, the challenges more pronounced in case of distribution systems. Basic reason behind these huge power losses is resistive loss, as distribution systems are operated at much lower voltages compared to transmission systems. Hence, operating current in distribution system is much more than that in transmission systems and hence, larger power loss (resistive) in distribution systems compared to transmission systems.

Load flow is an important tool for the analysis of any power system and it is used in the operational as well as planning stages. The recent tendency towards the distribution automation (DA) has led researchers to develop the so-called control functions, which perform on-line predefined tasks, either in emergency or normal conditions. These application programs require robust and efficient load flow solution method. At present, power system engineers are forced to place more emphasis on reducing losses at the distribution level. Current social, political and economic trends place a cap on the expansion of the generation and transmission levels.

Berg *et al*. [1] presented a backward method. It works by using backward procedure to update the equivalent impedance at the sending end. This method is very costly and quite sensitive to the system load level and load distribution, as well as the system structures. Baran *et al*. [3] presented a forward method. However, this method still has disadvantages. Oriented from ladder network concepts, the 'branch flow equations' should essentially solved by a Newton-Raphson approach. Nanda *et al.* [4] presented a new backward algorithm. This method uses the backward sweep to calculate the sending end voltage. The disadvantage of this method is the difficulty of the convergence because the backward calculation of the voltage drop usually causes the voltage of the sending end to go very high compared to the setting value. Hence, a special value is needed to control the voltage profile during iterations in terms of the convergence tolerance. The authors did not explain how to determine these special values, theoretically or heuristically. Afsari *et al.* [5] proposed the algorithm for the branch-bound solution of a distribution network. It uses a backward sweep method where the voltage of the

*Manager, Essar Steel India Ltd, 27th km, Surat Hazira Road, Hazira - 394270, Gujarat, India. E-mail: Tejas.Vyas@essar.com
**Associate Prof., Department of Electrical Engineering, SVNIT, Surat, Gujarat-395007. E-mail: rr@eed.svnit.ac.in

terminal node on the main feeder must be known to start the solution. The convergence pattern of the algorithm depends largely on the initial voltage of the terminal node. Luo *et al*. [6] presented compensation method for weakly meshed networks. This method started from a network structure analysis to find the interconnection points. Then it breaks these interconnection points using the compensation method so that the meshed system structure could be changed to simple tree-type radial system. Haque [7] presented approach for meshed networks with more than one feeding node. The method first converts the multiple-source mesh network into an equivalent single-source radial type network by setting dummy nodes for the break points at distributed generators and loop connecting points. Then the traditional ladder network method can be applied for the equivalent radial system. Ghosh *et al.* [8] proposed a method that involves only the evaluation of a simple algebraic expression of voltage magnitudes and no trigonometric functions as opposed to the standard load flow case. Thus, computationally, the proposed method is very efficient and it requires less computer memory. Biswarup Das [9] has presented a method for radial distribution system power flow based on interval arithmetic. In this study, the load at different buses are assumed to vary over intervals. The load flow algorithm chosen is essentially a backward sweep/ forward sweep algorithm. However, the real arithmetic calculations in the algorithm have been replaced by complex interval arithmetic calculations. Udomask *et al.* [10] presents the proposed load transfer technique to solve slow convergence of power flow calculation due to additional load installation. The overall computational speed is numerically shorter than that of solving the problem without applying the load transfer technique. Power flow calculation is done by Newton-Raphson method. Bompard *et al.* [11] presented a study on the convergence characteristics of the backward/forward sweep method. Some useful indicators are introduced to estimate the number of iterations required to reach the convergence of the iterative process under a given tolerance. Augugliaro *et al.* [12] had developed a methodology for the analysis of radial or weakly meshed distribution systems supplying voltage dependent loads . The solution process is iterative and, at each step, loads are simulated by means of impedances. Leandro *et al.* [13] presented comparisons of two power flow methodologies for distribution system analysis: the three-phase current injection method – TCIM and the forward/backward sweep – FBS. Augugliaro *et al.* [14] presented a new backward/forward (b/f) methodology for the analysis of distribution systems with constant power loads. The developed method, although deriving conceptually from the backward forward methodology, presents only the backward phase in which all the network variables are evaluated considering a scaling factor. Ulas *et al.* [15] had presented method that takes into account voltage dependency of static loads, and line charging capacitance. The method is based on the forward and backward swap. Belal Mohammadi *et al.* [16] presented a fuzzy approach load flow for balanced and unbalanced radial distribution systems with incorporating load model. Tripathy *et al.* [17] had presented an application of mathematician K.M. Brown's method to load-flow solution. The method is particularly effective for the solution of ill-conducted systems of nonlinear algebraic equations. Iwamoto *et al.* [18] developed, a load flow calculation method for ill-conditioned power system.The proposed method is very simple, has no mathematical approximations, and requires almost no additional storage and computation time when incorporated into the normal Newton-Raphson program. Fan Zhang *et al.* [19] derived a modified Newton method for radial distribution systems in which the Jacobian matrix is in UDUT form.. With this formulation, the conventional steps of forming the Jacobian matrix, LU factorization and forward/back substitution are replaced by back/forward sweeps on radial feeders with equivalent impedances. Costa *et al.* [20] described a sparse Newton Raphson formulation for the solution of the power flow problem, comprising 2n current injection equations written in rectangular coordinates. Paulo *et al.* [21] presented a new sparse formulation for the solution of unbalanced three-phase power systems using the Newton–Raphson method. The three-phase current injection equations are written in rectangular coordinates resulting in an order 6n system of equations. Lin *et al.* [22] introduced a solution on a current-injection based Newton–Raphson method in rectangular coordinates. The “Fast Decoupled” idea was incorporated into the distribution network analysis for first time. Teng [23] introduced a method that is based on idea of modified Gauss-Seidel algorithm to the implicit Z-bus gauss method, a three-phase distribution network can be separated into three single-phase distribution networks and can be solved phase by phase. Carol *et al.* [24] presented a three-phase power flow solution method for real-time analysis of primary distribution systems. This method is a direct extension of the compensation-based power flow method for weakly meshed distribution systems from single phase to three phase, with the emphasis on modeling of dispersed generation (PV nodes), unbalanced and distributed loads, and voltage regulators and shunt capacitors with automatic local tap controls. Teng [25,26] developed a network-topology-based three-phase distribution power flow algorithm. Two developed matrices are enough to obtain the power flow solution: they are the bus-injection to branch-current matrix and the branch-current to bus-voltage matrix. Arvindhababu *et al.* [27] presented a novel technique to obtain the solution of load flow in radially operated distribution networks. This method is simple, easy to program and is based on the formation of a constant sparse upper triangular matrix, which is used to determine the bus voltages. In method presented by Halue [28] The load flow problem of a distribution system was also formulated in terms of three sets of recursive equations so that very sophisticated voltage dependent load models can easily be incorporated in these equations. Ranjan *et al.* [2] had presented a simple and efficient algorithm to solve radial distribution networks. The algorithm uses the basic principle of circuit theory and can be easily understood. Das *et al.* [29] presented a novel method for solving radial distribution networks. The radial feature of the network has been fully exploited to develop an algorithm by a unique lateral, node and branch numbering scheme. The proposed method involves only the evaluation of simple algebraic voltage expressions without any trigonometric functions. But in this method, they have used algorithm to calculate node beyond branch that is very complicated and time consuming.

In this paper main aim of the author is to prepare efficient method to find out power flow of distribution network by using simple algebraic equations with reduced calculation time. The general algorithm consists of two basic steps: forward sweep and backward sweep. The forward sweep is mainly a voltage drop calculation from the sending end to the far end of a feeder or a lateral; and the backward sweep is primarily a current summation based on the voltage updates from the far end of the feeder to the sending end. Then by using KVL and KCL, the voltage drop can be obtained. This proposed method can easily include composite load modeling if composition of load is known. Several distribution networks has been solved efficiently using the proposed method with high accuracy. The time consumed in calculation of load flow by using proposed method is also very less than other methods.

2.0 POWER FORMULATION OF DISTRIBUTION NETWORK

The electrical equivalent of a typical branch of distribution network is shown in Figure 1.

The current through the branch jj is given by From Figure 1, we have the following equations:

$$I(1) = \frac{|V(1)| < \delta - |V(2)| < \delta(2)}{R(1) + jX(1)} \tag{1}$$

$$P(2) - jQ(2) = V^*(2)\, I(1) \tag{2}$$

From equations (1) and (2) we have

$$I(1) = \frac{P(2) - jQ(2)}{V^*(2)} \tag{3}$$

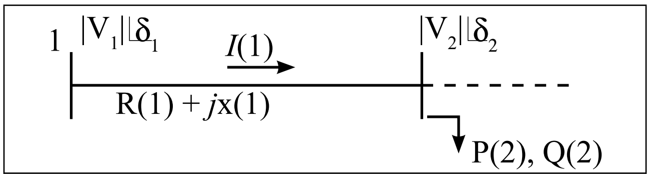

FIG. 1 ELECTRICAL EQUIVALENT OF A TYPICAL BRANCH

From equations (2) and (3) we obtain:

$$\frac{|V(1)| < \delta - |V(2)| < \delta(2)}{R(1) + jX(1)} = \frac{P(2) - jQ(2)}{V^*(2)} \quad (4)$$

Therefore,

$|V(1)|\ |V(2)| < \delta(1) - \delta(2) - |V(2)|^2 = [P(2) - jQ(2)]\ [R(1) + jX(1)]$ (5)

$|V(1)|\ |V(2)| \cos[\delta(1) - \delta(2)] - |V(2)|^2 + j|V(1)|\ |V(2)| \sin[\delta(1) - \delta(2)] = [P(2)R(1) + Q(2)X(1)] + j[P(2)X(1) - Q(2)R(1)]$ (6)

$$|V(2)| = \{|(P(2)R(1) + Q(2)X(1) - 0.5|V(1)|^2)^2 - (R^2(1) + X^2(1))\ (P^2(2) + Q^2(2))]^{1/2} - (P(2)R(1) + Q(2)X(1) - 0.5|V(1)|^2)\}^{1/2} \quad (7)$$

where P(2) and Q(2) are total real and reactive power loads fed through node 2.

(P^2) = sum of the real power loads of all the nodes beyond node 2 plus the real power load of node 2 itself plus the sum of the real power losses of all the branches beyond node 2.

(Q^2) = sum of the reactive power loads of all the nodes beyond node 2 plus the reactive power load of node 2 itself plus the sum of the reactive power losses of all the branches beyond node 2. Equation (6) can be written in generalized form:

$$|V(m^2)| = [B(j) - A(j)]^{1/2} \quad (8)$$

Where,

$$A(j) = P(m^2)^*R(j) + Q(m^2)^* \times (j) - 0.5^*|V(m^1)|^2 \quad (9)$$

$$B(j) = \{A^2(j) - [R^2(j) + x^2(j)]^* \times [P^2(m^2) + Q^2(m^2)]\} \quad (10)$$

j is the branch numberml=IS(j) and m^2=IR(j). Real and reactive power losses in branch 1 can be given by

$$LP(1) = \frac{R(1)^*[P^2(2) + Q^2(2)]}{|V(2)|^2} \quad (11)$$

$$LQ(1) = \frac{X(1)^*[P^2(2) + Q^2(2)]}{|V(2)|^2} \quad (12)$$

$$LP(j) = \frac{R(j)^*[P^2(m^2) + Q^2(m^2)]}{|V(m^2)|^2} \quad (13)$$

$$LQ(j) = \frac{X(j)^*[P^2(m^2) + Q^2(m^2)]}{|V(m^2)|^2} \quad (14)$$

For the same line, given in Figure 1, Kirchoff's Voltage Law (KVL) can be written as:

$$V_{m2} = V_{m1} - I(jj)^*Z(jj) \quad (15)$$

$$T_{LP} = \sum_{jj=1}^{L_{N1}} LP_{(jj)} \quad (16)$$

3.0 SOLUTION METHODOLOGY

It is assumed that the three-phase radial distribution networks are balanced and can be represented by their equivalent single-line diagrams. To start, flat voltage profile is assumed at each node.

Figure 2 shows single-line diagram of a distribution feeder. The branch number sending-end and receiving-end node and nodes beyond branches of this feeder are given in Table 1. Consider branch 1, the receiving-end node voltage can be written as

$$V(2)=V(1)-I(1)^*Z(1) \quad (17)$$

Similarly for branch 2,

$$V(3)=V(2)-I(2)^*Z(2) \quad (18)$$

As the substation voltage and load are known, so if I(1) is known, i.e. current of branch 1, it is easy to calculate V(2) from eqn. (17). Once V(2) is known, it is easy to calculate V(3) from eqn. 2, if the current through branch 2 is known. Similarly, voltages of nodes 4,5,...,n can be find out. In generalized form voltage equation for receiving end node is given in eqns. (19).

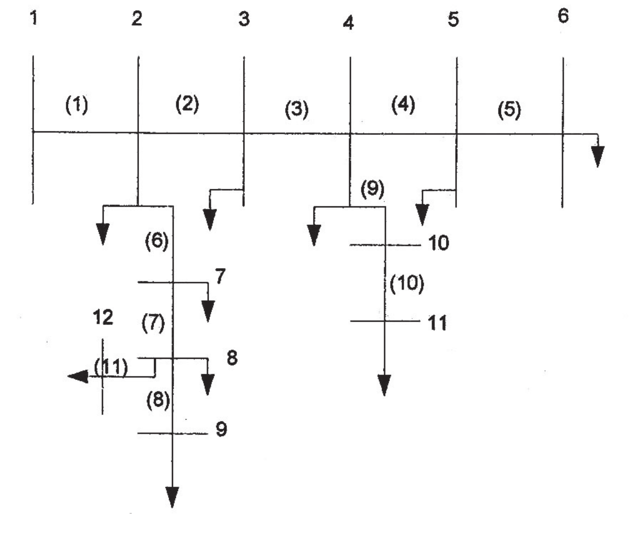

FIG. 2 SINGLE-LINE DIAGRAM OF RADIAL DISTRIBUTION NETWORK

TABLE 1

BRANCH NUMBER, SENDING END NODE, RECEIVING END NODE, AND NODES BEYOND BRANCHES OF FIGURE 2

Branch Number (jj)	**Sending End Node IS(jj)**	**Receiving End node IR(jj)**	**Nodes beyond branch**	**Total number of nodes beyond branch jj**
1	1	2	2,3,7,8,4, 5,10,9, 12,6,11	1
2	2	3	3,4,5,10, 6,11	6
3	3	4	4,5,10, 6,11	5
4	4	5	5,6	2
5	5	6	6	1
6	2	7	7,8,9,12	4
7	7	8	8,9,12	3
8	8	9	9	1
9	4	10	10,11	2
10	10	11	11	1

$$V(m^2) = V(rn1) - I(jj)\,Z(jj) \quad (19)$$

where jj is the branch number.

$$m^2=IR(jj) \quad (20)$$

$$m^1=IS(jj) \quad (21)$$

Eqn can be evaluated for jj=1,2,..., LN1 (LN1=NB-1=number of branches, NB=number of nodes). Current through branch 1 is equal to the sum of load currents of all the nodes beyond branch 1 plus the sum of charging currents of all the nodes beyond branch 1, i.e.

$$I(1)=\sum_{i=2}^{LN1} IL(i)+\sum_{i=2}^{LN1} IC(i) \quad (22)$$

The current through branch 2 is equal to the sum of the load currents of all the nodes beyond branch 2 plus the sum of the charging currents of all the nodes beyond branch 2, i.e.

$$I(2) = IL(3) + IL(4) + IL(5) + IL(6)+ IL(10) + IL(11) + IC(3) + IC(4) + IC(5) + IC(6) +IC(10)+IC(11) \quad (23)$$

Therefore, if it is possible to identify the nodes beyond all the branches, it is possible to compute all the branch currents. Identification of the nodes beyond all the branches is realized through an algorithm as explained in upcoming Section.

The load current of node i is

$$IL(i)=\frac{PL(i)-jQL(i)}{V^*(i)} \quad i = 2,3,........., NB \quad (24)$$

The charging current at node i is

$$IC(i) = y_0(i)*V(i) \quad i = 2,3, ……, NB \quad (25)$$

Load currents and charging currents are computed iteratively. Initially, a flat voltage of all the nodes is assumed and load currents and charging currents of all the loads are computed using eqns. (24) and 25. A detailed load-flow-calculation procedure is described in section.

The real and reactive power loss of branch jj are given by:

$$LP(jj) = |I(jj)|^2*R(jj) \quad (26)$$

$$LQ(jj) = |I(jj)|^2 * X(jj) \tag{27}$$

4.0 IDEBTIFICATION OF NODES BEYOND ALL THE BRANCHES

Before the detailed algorithm is given, the details of the methodology of identifying the nodes beyond all branches will be discussed. This will help in finding the exact current flowing through all the branches.

N=1,2,3….., LN1(n indicates branch of Figure 2, refer Table 1) k is the node count (identifies number of nodes beyond a particular branch); u is the node identifier (helping to identify nodes beyond all the branches) and nb(i,j) is the matrix which stores the nodes beyond all the branches.

Step 1: First read sending and receiving end nodes, total no of bus, load connected to bus, base MVA, base kV and no of open switches.

Step 2: Create a square matrix nb of size n×n (n=total no of branch) and initialize it with zero.

Step 3: Create a loop variable i=1,2,3,…., n. Function of i is to rotate loop until total no of rows of matrix nb executed.

Step 4: Initialize first variable of every row of matrix nb with receiving node of related branch.

Step 5: Create two variables m and j and initialize it with zero. Where m indicates the next position in the node matrix nb and j is for breaking the inner loop(u) if no. of columns has reached.

Step 6: After that, create a loop variable u=1,2,3,….., n; Function of u is to rotate loop until total no of columns of matrix nb executed.

Step 7: Create a loop variable k=1,2,3, ….., n; Function of k is to rotate loop to identify nodes beyond branches. For value of i=1 and u=1, check if any place in nb(1,1)=2 is given as sending node for every branch. If in any branch value sending node equals to nb(1,1) assign receiving node of that branch to nb(1,2). Again if any branch value sending node equals to nb(1,1), assign receiving node of that branch to nb(1,3). This will continue for all branches of the loop value of k=1,2,3,…, n. Same way, it will check for the value of i=1 and u=2,3, ….., n. and it will find next values of first row of nb matrix. In between if the value of j exceeds the total no of columns, then j will check it and break the loop. For other branches, it will check for the value of i=2,3,…, n. and u=1,2,3,….., n. for the other elements of matrix nb.

The calculated matrix nb will contain no of nodes beyond the branches. This will help in computing the current flowing through all the branches.

Flow chart for identification of nodes beyond particular branch is given in Figure 3.

5.0 LOAD FLOW CALCULATIONS

Once all nodes beyond each branch are identified, then calculate the current flowing through each branch. For this purpose, the load current and charging current of each node are calculated by using eqns. (24) and (25). Total branch current is summation of load current and charging current of that branch.

$$I(jj)=IL\{IE(jj,i)\}+IC\{IE(jj,i)\} \tag{28}$$

Branch current is given by eqn. (28). The voltage of each node is then calculated by using (19). Real and reactive power loss of each branch is calculated by using (26) and (27), respectively. Once the new values of the voltages of all the nodes are computed, convergence of the solution is checked. If it does not converge, then the load and charging currents are computed using the most recent values of the voltages and the whole process is repeated.

Losses is given in Figure 4. The convergence criterion of the proposed method is that if, in successive iterations, the maximum difference in voltage magnitude (DVMAX) is less than 0.0001 p.u., then the solution has then converged.

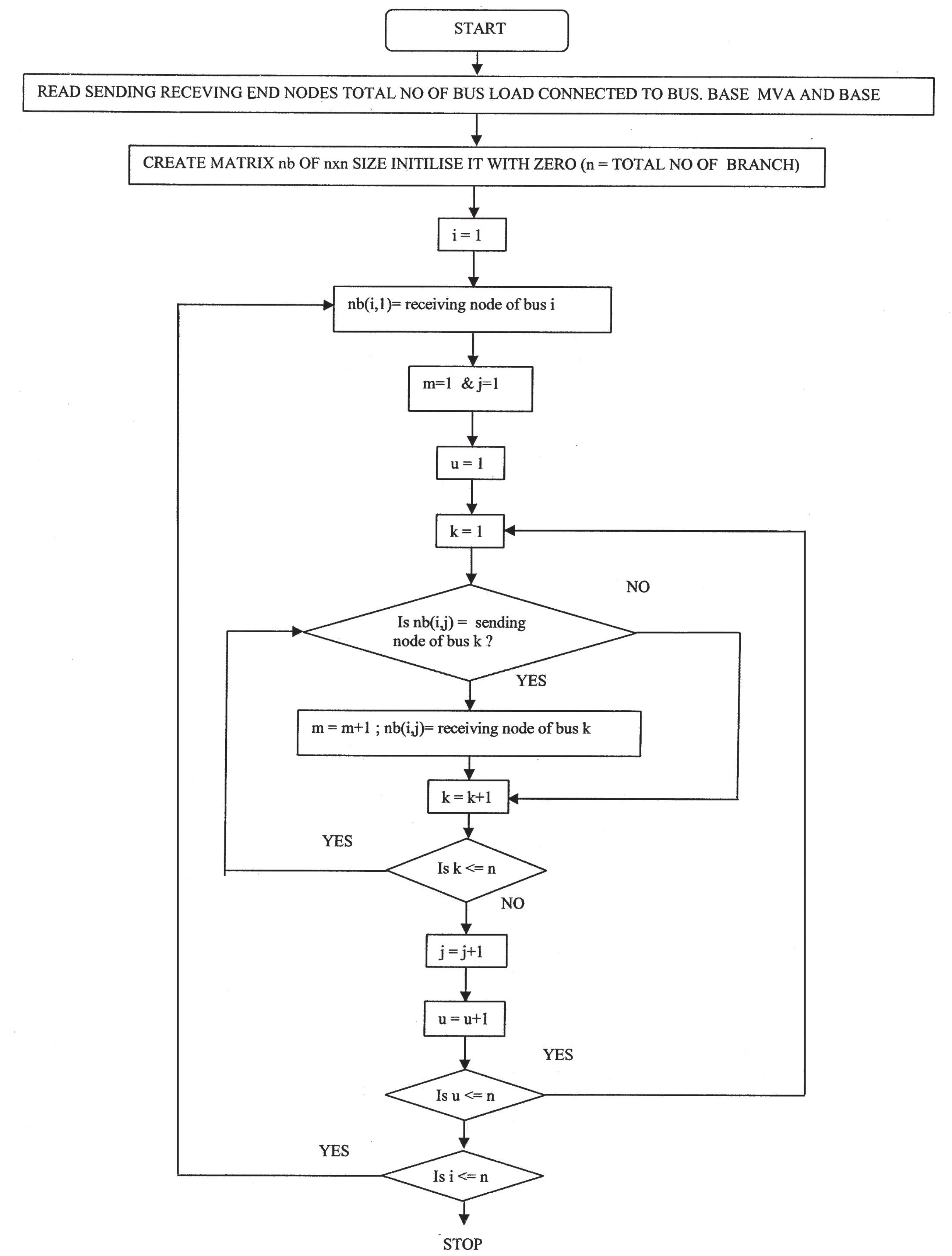

FIG. 3 FLOWCHART FOR IDENTIFYING NODES BEYOND ALL THE BRANCHES

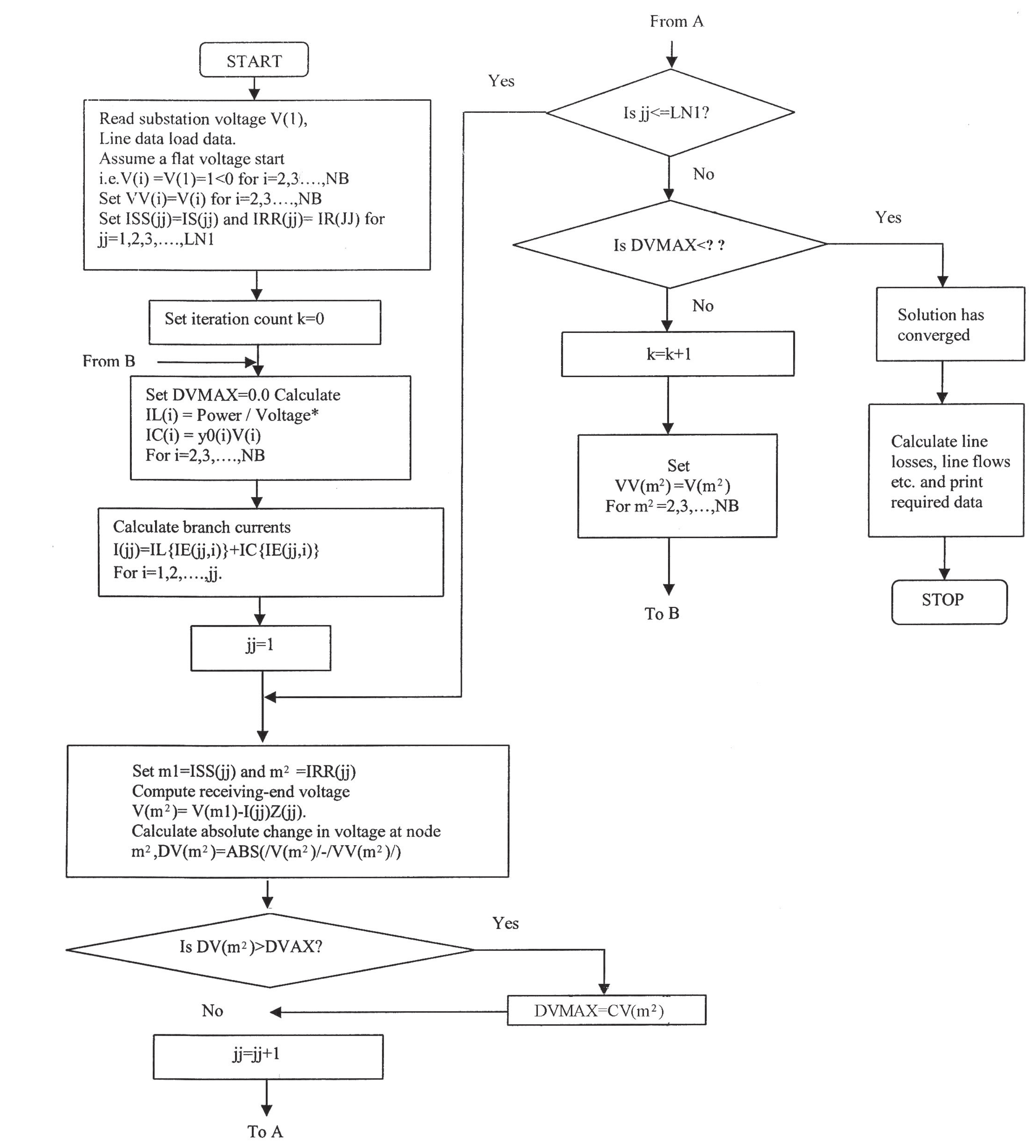

FIG. 4 FLOWCHART FOR LOAD-FLOW CALCULATION OF RADIAL DISTRIBUTION NETWORK

6.0 CASE STUDIES

To demonstrate the effectiveness of the proposed method, five test cases are selected. The first test case is a 33-node distribution feeder. The second, third, fourth and fifth test cases are for 69, 16, 117 and 68-node distribution feeder, respectively. Data for 33 node network is given in Appendix 1. Table 2A gives the load-flow results of 33 node distribution network. Tables 2C, 3–6 gives the real-power loss, reactive power, minimum voltage magnitude for each example and number of iteration cycle used. In Figures 5–7 graph of iteration verses minimum voltage, power loss and time respectively.

TABLE 2A

LOAD-FLOW SOLUTION OF 33-NODE RADIAL DISTRIBUTION NETWORK

Node number	Voltage magnitudes (p.u)		Error in %
	Proposed method	Existing method[30]	
1	1.00000	1.00000	0.000000
2	0.99701	0.99703	0.000020
3	0.98288	0.98289	0.000010
4	0.97537	0.97538	0.000010
5	0.96794	0.96796	0.000021
6	0.94947	0.94948	0.000011
7	0.94594	0.94595	0.000011
8	0.93229	0.93230	0.000011
9	0.92595	0.92597	0.000022
10	0.92010	0.92009	−0.000011
11	0.91923	0.91922	−0.000011
12	0.91771	0.91771	0.000000
13	0.91154	0.91153	−0.000011
14	0.90925	0.90924	−0.000011
15	0.90782	0.90782	0.000000
16	0.90644	0.90643	−0.000011
17	0.90439	0.90439	0.000000
18	0.90378	0.90377	−0.000011
19	0.99648	0.99650	0.000020
20	0.99290	0.99292	0.000020
21	0.99220	0.99221	0.000010
22	0.99156	0.99158	0.000020
23	0.97929	0.97931	0.000020
24	0.97262	0.97264	0.000021
25	0.96930	0.96931	0.000010
26	0.94754	0.94755	0.000011
27	0.94497	0.94499	0.000021
28	0.93353	0.93354	0.000011
29	0.92531	0.92532	0.000011
30	0.92175	0.92177	0.000022
31	0.91759	0.91760	0.000011
32	0.91668	0.91640	−0.000306

TABLE 2B

LOAD-FLOW SOLUTION OF 33-NODE RADIAL

Itera-tion	Min voltage	Total loss kW	Total	Time
1	0.91144	0.182188	0.123256	0.027898
2	0.90434	0.208611	0.141471	0.033102
3	0.90375	0.210808	0.142994	0.033508
4	0.9037	0.210988	0.143119	0.034396

TABLE 2C

TOTAL POWER LOSS, REACTIVE POWER AND MINIMUM VOLTAGE MAGNITUDE, NO OF ITERATION

Quantity	33-node Distribution network	
	Proposed method	Existing method[30]
Real-power loss (kW)	210.972	210.998
Reactive power loss (kVAr)	143.117	143.032
Minimum voltage magnitude (p.u)	At node-18 =0.903764	At node-18 =0.90377
No of iteration cycles	4	–
Time required for execution(s)	0.0331	–

TABLE 3

TOTAL POWER LOSS, REACTIVE POWER AND MINIMUM VOLTAGE MAGNITUDE, NO OF ITERATION

Quantity	69-node Distribution network	
	Proposed method	Existing method[30]
Real-power loss (kW)	224.999	224.960
Reactive power loss (kVAr)	102.166	102.147
Minimum voltage magnitude (p.u)	At node-65 =0.90918	At node-65 =0.90919
No of iteration cycles	5	–
Time required for execution(s)	0.0378	–

TABLE 4

TOTAL POWER LOSS, REACTIVE POWER AND MINIMUM VOLTAGE MAGNITUDE, NO OF ITERATION

Quantity	16-node distribution network	
	Proposed method	**Existing method[30]**
Real-power loss (kW)	511.398	511.4
Reactive power loss (kVAr)	590.325 =0.969269	–
Minimum voltage magnitude (p.u)	At node-12	–
No of iteration cycles	3	–
Time required for execution(s)	0.01638	–

TABLE 5

TOTAL POWER LOSS, REACTIVE POWER AND MINIMUM VOLTAGE MAGNITUDE, NO OF ITERATION

Quantity	117-node distribution network	
	Proposed method	**Existing method[3]**
Real-power loss (kW)	1296.548	1294.3
Reactive power loss (kVAr)	978.047	–
Minimum voltage magnitude (p.u)	At node-80=0.868800	At node-80=0.869800
No of iteration cycles	5	–
Time required for execution(s)	0.0558	–

TABLE 6

TOTAL POWER LOSS, REACTIVE POWER AND MINIMUM VOLTAGE MAGNITUDE, NO OF ITERATION

Quantity	69-node Distribution network	
	Proposed method	**Existing method[30]**
Real-power loss (kW)	337.98	337.45
Reactive power loss (kVAr)	302.166	–
Minimum voltage magnitude (p.u)	At node-67=0.88487	At node-67=0.88389
No of iteration cycles	5	–
Time required for execution(s)	0.0382	–

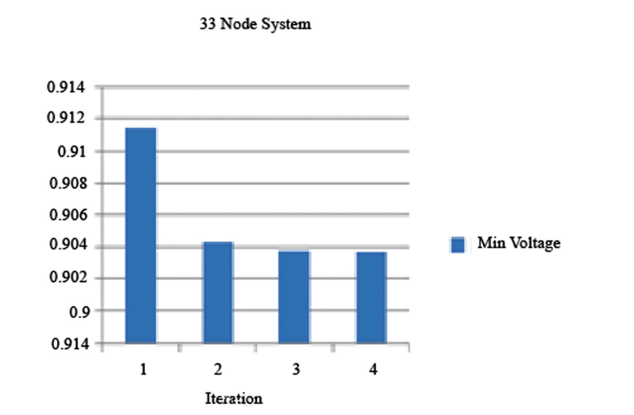

FIG. 5 GRAPH OF MINIMUM VOLTAGE VERSES ITERATION FOR 33 NODE SYSTEM

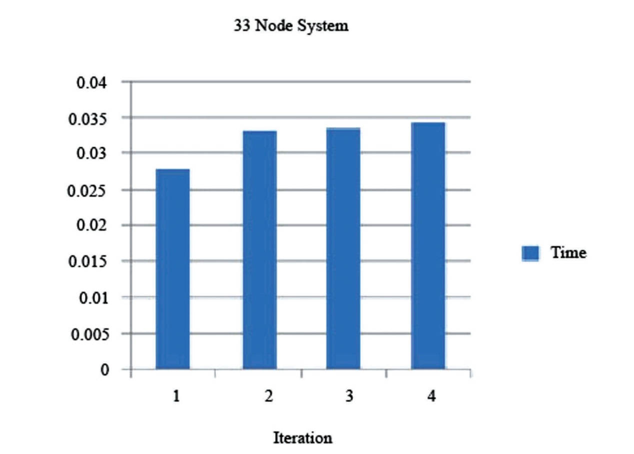

FIG. 7 GRAPH OF ITERATION TIME VERSES ITERATION NUMBER FOR 33 NODE SYSTEM

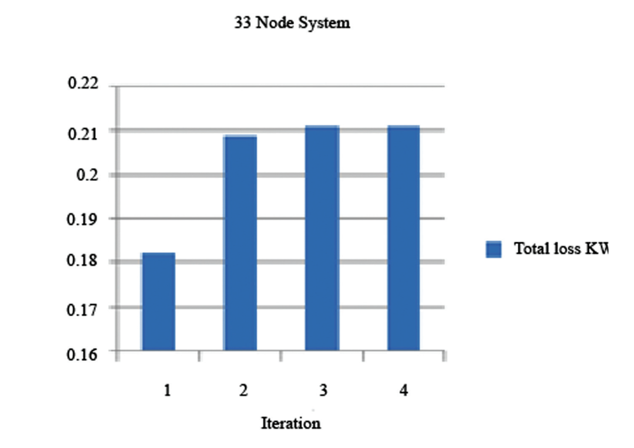

FIG. 6 GRAPH OF TOTAL LOSS IN kW VERSES ITERATION FOR 33 NODE SYSTEM

7.0 CONCLUSIONS

A simple and efficient load-flow technique has been proposed for solving ill-conditioned radial distribution networks. The method is based on solution of simple algebraic equation. It completely exploits the radial feature of the distribution network. The proposed method always guarantees convergence of any type of practical radial distribution network with a high R/X ratio. The method is tested successfully for four test cases. This method is good and fast compared to existing methods. The proposed method has been simulated in Matlab 8.0. Several other networks have also been tested by using this method.

APPENDIX

LINE DATA AND LOAD DATA OF TEST CASE 1: 33-NODE RADIAL DISTRIBUTION NETWORK

Branch number	Branch		Branch impedance		Charging admitance (Y)	Load	
	Sending end code	Receiving end node	r(Ohm)	x(Ohm)		P(kW)	Q(kVAr)
1	1	2	0.0922	0.0477	0	100	60
2	2	3	0.4930	0.2511	0	90	40
3	3	4	0.3660	0.1864	0	120	80
4	4	5	0.3811	0.1941	0	60	30
5	5	6	0.8190	0.7070	0	60	20
6	6	7	0.1872	0.6188	0	200	100

7	7	8	1.7114	1.2351	0	200	100
8	8	9	1.0300	0.7400	0	60	20
9	9	10	1.0400	0.7400	0	60	20
10	10	11	0.1966	0.0650	0	45	30
11	11	12	0.3744	0.1238	0	60	35
12	12	13	1.4680	1.1550	0	60	35
13	13	14	0.5416	0.7129	0	120	80
14	14	15	0.5910	0.5260	0	60	10
15	15	16	0.7463	0.5450	0	60	20
16	16	17	1.2890	1.7210	0	60	20
17	17	18	0.7320	0.5740	0	90	40
18	2	19	0.1640	0.1565	0	90	40
19	19	20	1.5042	1.3554	0	90	40
20	20	21	0.4095	0.4784	0	90	40
21	21	22	0.7089	0.9373	0	90	40
22	3	23	0.4512	0.3083	0	90	50
23	23	24	0.8980	0.7091	0	420	200
24	24	25	0.8960	0.7011	0	420	200
25	6	26	0.2030	0.1034	0	60	25
26	26	27	0.2842	0.1447	0	60	25
27	27	28	1.0590	0.9337	0	60	20
28	28	29	0.8042	0.7006	0	120	70
29	29	30	0.5075	0.2585	0	200	600
30	30	31	0.9744	0.9630	0	150	70
31	31	32	0.3105	0.3619	0	210	100
32	32	33	0.3410	0.5302	0	60	40

REFERENCES

[1] Jr. Berg R, Hawkins E S and Pleines W W. 'Mechanized calculation of unbalanced load flow on radial distribution circuits', *IEEE, Trans. Power Apparatus and Systems,* Vol. 86, No. 4, 1967.

[2] Shirmohammadi D. "A compensation-based power flow method for weakly meshed distribution and transmission networks", *IEEE, Trans. Power Syst.,* Vol. 3, pp. 753–762, 1988.

[3] Baran M and Wu F F. "Optimal sizing of capacitors placed on a radial distribution system", *IEEE, Transactions Power Delivery*, Vol. 4, pp. 1105–1117, 1989.

[4] Nanda J, Srinivas M S, Sharma M, Dey S S and Lai L L. "New findings on radial distribution system load flow algorithms",

Proc. IEEE, PES Winter Meeting, Singapore, pp. 1157–1161, 2000.

[5] Afsari M, Singh S P, Raju G S. "A fast power flow solution of radial distribution networks", *Electric Power Components and System*, Vol. 30, pp. 1065–1074, 2002.

[6] Luo G X and Semlyen A. "Efficient load flow for large weakly meshed networks", *IEEE, Transactions Power Systems*, Vol. 5, pp. 1309–1316, 1990.

[7] Haque M H. "A general load flow method for distribution system", *Electric Power System Research*, Vol. 54, pp 47–54,2000.

[8] Gosh S and Das D. "Method for load-flow solution of radial distribution networks" *IEE, Proc. Generation Transmission Distribution*, Vol.146, pp. 641–648, 1999.

[9] Das B. "Radial distribution system power flow using interval arithmetic", *Electric Power and Energy System*, Vol. 24, pp. 827–836, 2002.

[10] Udomask T, Padej Pao-La-Or and Kulworawanichpong T. "Application of load transfer technique for distribution power flow analysis", *World Academy of Science, Engineering and Technology*, Vol.47, 2008.

[11] Bompard E, Carpaneto E, Chicco G, Napoli R. "Convergence of the backward/forward sweep method for the load flow analysis of radial distribution systems", *Electrical power and Energy Systems*, Vol. 22, pp. 521–530, 2000.

[12] Augulioro A, Dusonchet L, Favuzza S, Ippolito M G, Riva Sanseverino E. "A backward sweep method for power flow solution in distribution networks", *Electrical power and Energy System*, Vol. 32, pp. 271–280, 2010.

[13] Leandro Ramos de Araujo, Debora Rosana Ribeiro penido, Sandoval Carneiro Junior, Jose Luiz Rezende Pereira, Paulo Augusto Nepomuceno Garcia, "Comparisons between the three-phase current injection method and the forward/backward sweep method", *Electrical Power and Energy Systems*, Vol. 32, pp. 825–833, 2010.

[14] Augulioro A, Dusonchet L, Favuzza S, Ippolito M G, Riva Sanseverino E. "A new backward/forward method for serving radial distribution networks with PV nodes", *Electrical Power Systems and Research*, Vol. 78, pp. 330–336, 2008.

[15] Ulas Eminoglu, Haken Hocaoglu M. "A new power flow method for radial distribution systems including voltage dependent load models", *Electrical Power Systems Research*, Vol. 76, pp. 106–114, 2005.

[16] Belal Mohammadi Kalesar, Ali Reza Seifi. "Fuzzy load flow in balanced and unbalanced radial distribution systems incorporating composite load model", *Electrical Power System Research*, Vol. 32. pp. 17–23, 2010.

[17] Tripathy S C, Purge Prasad G S S S K. "Load flow solution for in-conditioned Power systems by Luadratically convergent Newton-like method", *IEE Proc.*, Vol. 127, pp. 273–281, September 1980.

[18] Iwamoto S, Tamura Y. "A load flow calculation method for ill-conditioned power systems". *IEEE, Transaction on Power Apparatus and Systems*, Vol-PAS-100, 1981.

[19] Fan Zhang, Carol S Cheng. "A modified newton method for radial distribution System Power flow Analusis" *IEEE, Transaction on Power Systems,* Vol. 12, pp. 389–398, 1997.

[20] Vander Menengoy da Costa, Nelson Martins, Joes Luiz R. Pereira. "Developments in the newton raphson power flow formulation based on current injection", *IEEE, Transaction on Power Systems,* Vol. 14, pp. 1320–1327, 1999.

[21] Paulo A N Gracia, Jose Luzi R, Pereira, Sandoval Carneiro, Vander M da and Costa, Nelson Martins. "Three-phase Power flow alculations using the current injection method", *IEEE, Transaction on Power Systems,* Vol. 15, pp. 508–515, 2000.

[22] Whei- Min Lin, Jen-Hao Teng. "Three-phase distribution network fast decoupled power flow solutions", *Electrical Power and Energy System*, Vol. 22, pp. 375–380, 2000.

[23] Jen Hao Teng, "A modified Gauss-seidal algorithm of three-phase power flow analysis in distribution networks", *Electrical Power and Energy Systems*, Vol. 24, pp. 97–102, 2002.

[24] Carol S. Cheng. Darium shirmohammadi. "A three-phase flow method for real time distribution system analysis", *IEEE, Transaction on Power Systems*, Vol. 10, pp. 671–680, 1995.

[25] JEN-Hao-Teng. "A network-topology-based 3-phase load flow for distribution systems", *Proc. Natl,. Sci. Counc. (ROCCA)*, Vol. 24, pp. 259–264, 2000.

[26] JEN-Hao-Teng. "A direct approach for distribution system load flow solutions", *IEEE, Transactions on Power Delivery*, Vol. 18, pp. 882–888, 2003.

[27] Aravindhababu P, Ganpathy S, Nayar K R. "A novel technique for the analysis of radial distribution systems", *Electrical Power and Energy Systems*, Vol. 23, pp. 167–171, 2001.

[28] Halue M H. "Load flow solution of distribution systems with voltage dependent load models", *Electrical Power Systems Research*, Vol. 36, pp. 151–156, 1996.

[29] Das D, Nagi H S, Kothari D P. "Novel method for serving radial distribution networks", *IEE, Proc. Gener. Transm. Distrib.*, Vol. 141, pp. 291–299, 1994.

[30] Ranjan R, Das D. "Simple and Efficient computer algorithm to solve radial distribution networks", *Electrical Power Components and Systems*, Vol. 31, pp. 95–107, 2003.