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Stack Optimization of Thermoacoustic Refrigerator

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The performance of the thermoacoustic refrigerator depends upon a very large number of parameters, and hence the components are optimized using linear thermoacoustic theory to restrict the number of variables by dimensionless normalization technique. Since the stack is considered as the heart of the thermoacoustic refrigerator, the stack design parameters are the most significant parameters for the optimal overall performance. The stack optimization result shows that a decrease in stack length and center position from the loud speaker increases stack performance. Determination of optimum stack length and center position helps in designing the refrigerator and to provide adequate space for instrumentation in practical work. The cross-sectional area of the stack and hence the stack diameter is calculated for the required cooling load capacity which is considered as the basic parameter on which the design of other components depends. The effect of mean temperature of the gas has not received attention in the literature and hence the performance of the stack at higher mean temperature of the gas is theoretically evaluated. The improvement in the stack coefficient of performance COP_S *is reported compared to published experimental optimization studies at the design conditions considered in this paper, and the results are in good agreement with past established work.*

1.0 INTRODUCTION

Thermoacoustic refrigerator is an eco-friendly cooling device which uses standing sound waves generated by loudspeaker positioned at one end of resonator as an acoustic power input to pump heat. The heat transfer occurs between working gas and heat exchangers positioned at both ends of the stack assembled in a resonator as shown in Figure 1 and in which both ends of the resonator act as pressure antinodes. The velocity antinode is located in the middle. The pressure differential on both ends of the stack produces temperature difference as the parcel is moved by the standing wave due to compression and expansion. The principles of the thermoacoustic refrigerator can be found in many sources [1,8]. Tijani *et al*. [5,6] have described not only the designing procedure of constructing the device, but also its performance measurement. Although the thermoacoustic refrigerator can achieve a substantial fraction of Carnot's efficiency, the relative coefficient of performance (COPR) at present is 0.1–0.2 compared to 0.33–0.5 for conventional refrigeration [8]. Thermoacoustics is the most recently implemented heat pumping cycle and most of the devices built so far are for research purposes, and the achieved performance results are remarkable.

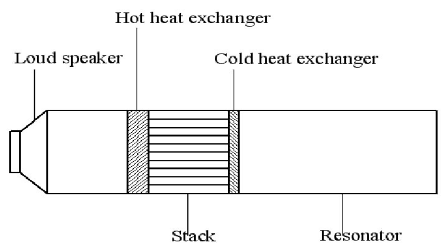

FIG. 1 COMPONENTS OF A THERMOACOUSTIC REFRIGERATOR

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In the literature [5], the design procedure for stack optimization is presented and the stack coefficient of performance (COP_s) is reported to be 1.3. In this paper, the theoretical calculations for improving the performance of the stack are presented by increasing the mean temperature of the gas which is due to attention in the present literature. The stack optimization of thermoacoustic refrigerator is discussed using the dimensionless heat flow and acoustic power flow equations. The COP_s is calculated for the various stack lengths and center positions at the operating and design conditions considered, and hence the stack is optimized at the higher performance values by considering practical reasons which restrict the performance.

2.0 THEORETICAL BACKGROUND

Stack is a kind of heat exchanger placed in the resonator which consists of many surfaces or plates parallel to the axis of the resonator tube. When the acoustical standing wave with high intensity exists in the resonator, heat transfer between stack plates and gas within resonator can occur. Because one end of the stack is in contact with the hot reservoir and the other end with the cold reservoir, the net effect is a heat transfer from the cold reservoir to the hot reservoir. The requirement of this heat transfer is that the temperature difference θ along the stack has to be less than a critical longitudinal temperature gradient θ_c, which is expressed by

$$\theta_C = (\gamma - 1)\ kLBT_m\ \text{cpt}\ (kX) \tag{1}$$

where $k=2\pi/\lambda$, known as wave number, L is the critical stack length, X is stack center position, B is stack porosity and T_m is mean temperature of gas. The normalized temperature gradient Γ is the ratio of the actual temperature gradient θ along the stack and the critical temperature gradient θ_c should clearly be less than one, which is the primary necessity for the device to function as a refrigerator. The expression for Γ is given by [1]

$$\Gamma = \frac{\theta \tan(kX)}{(\gamma - 1)BkLT_m} \tag{2}$$

In the design of thermoacoustic refrigerator, the following assumptions are made considering boundary layer and short-stack approximations [1].

- The length of the stack L is much smaller compared to the wavelength λ, so that the acoustic field is not significantly disturbed by the presence of stack.
- The thermal penetration depth δ_k is smaller than the half-stack spacing y_o. However, the viscous penetration depth δ_v does not feature in the analysis because the flow is assumed to be inviscid.
- The temperature difference across the stack ends θ is smaller than the mean temperature of the gas T_m, so that the thermophysical properties of the gas can be considered to remain constant.
- The thermal conductivity of the stack plate is neglected.
- The fluid friction at the inner walls of the resonator is neglected.
- The device functions at steady state and hence the mean temperature of the gas T_m and the temperature difference across the stack remain constant with time.

The other important parameters for the stack in thermoacoustic refrigerator are the porosity of the stack B, thermal penetration depth δ_k and viscous penetration depths δ_v expressed by

$$B = \frac{y_o}{y_o + l} \tag{3}$$

where y_o and l are half-stack plate spacing and half-stack plate thickness, respectively. The value for B is usually taken between 0.7 and 0.8, so that the acoustic field is not significantly disturbed.

$$\delta_k = \sqrt{\frac{2k}{\rho C_p \omega}} \tag{4}$$

and

$$\delta_v = \sqrt{\frac{2\mu}{\rho\omega}} \tag{5}$$

The viscous penetration depth describes the gas layer thickness near the stack plates in which the motion is resisted by viscous force. This gives a negative contribution to the thermoacoustic effect. However, as mentioned earlier, δ_v does not occur in the analysis. The thermal penetration depth determines the distance that heat can diffuse through a gas during time $1/\pi f$, and the optimum stack-plate spacing $2y_o$ is about four times thermal penetration depth δ_k [1,3,7]. The heat conduction through the stack material and gas in the stack region also has a negative effect, so that the stack must have low thermal conductivity and a heat capacity larger than the heat capacity of the working gas [1,4], hence the material Mylar is often chosen. The stack geometry may have parallel plates, circular pores, pin arrays, etc. The pin array stacks are the best, but they are too difficult to manufacture and hence parallel plate or circular plate stack is selected. Helium is used as working gas, since it has the highest sound velocity and is cheaper in comparison with other noble gases. The sound velocity u is calculated at mean temperature of the gas [4]. For the thermoacoustic refrigerator, the power density is proportional to the mean pressure P_m and the acoustic resonance frequency f [1], and hence it is favorable to choose P_m and f, as large as possible. These two parameters imply a stack with very small plate spacing but this makes construction difficult. Making compromise between these two effects, a 10 bar pressure and 400 Hz frequency, is chosen. In order to avoid nonlinear effects [1], a driving ratio D of less than 3 % is selected.

3.0 DESIGN OPTIMIZATION OF THE STACK

In this section, the design, analysis and the optimization of the thermoacoustic refrigerator stack are discussed. The design parameters are determined based on certain chosen operating parameters. The coefficient of performance of the stack, COP_s, defined as the ratio of the heat pumped by the stack to the acoustic power used by the stack, is to be maximized for the better performance of the device. The exact theoretical expression for COP_s is complicated because it contains large number of parameters and hence we have identified 18 parameters that include design and operating parameters and the parameters related to working gas and the stack. By means of normalization technique, the number of parameters has been reduced from 18 to 10 dimensionless independent variables by neglecting thermal conductivity of the stack. The parameters of paramount importance in thermoacoustic refrigerator design and the resultant normalized parameters are given an extra index n and are shown in Table 1. The expressions for the heat flow and acoustic power in a dimensionless form are given by [5]

$$Qn = -\frac{\delta_{kn} D^2 \sin 2X_n}{8\gamma(1+\sigma)} \times \left(\frac{\theta_n \tan(X_n) 1 + \sqrt{\sigma} + \sigma}{(\gamma-1)BL_n \ \ 1+\sqrt{\sigma}} - (1+\sqrt{\sigma} - \sqrt{\sigma\delta_{kn}}) \right) \tag{6}$$

and

$$W_n = \frac{\delta_{kn} L_n D^2}{4\gamma}(\gamma-1)B\cos^2 X_n \times \left(\frac{\theta_n \tan(X_n)}{BL_n(\gamma-1)(1+\sqrt{\sigma})\Lambda} - 1 \right) - \frac{\delta_{kn} L_n D^2}{4\gamma} \frac{\sqrt{\sigma}\sin^2 X_n}{B\Lambda} \tag{7}$$

where Λ is used as an intermediate variable and is defined as

$$\Lambda = 1 - \delta_{kn}\sqrt{\sigma} + \frac{\sigma\delta_{kn}^2}{2} \tag{8}$$

In Eqs. (6), (7) and (8), the data given in the Table 2 is used and in which all the thermo-physical properties of the gas are considered at mean temperature T_m. The performance of the stack, COP_s, is given by

$$COP_s = \frac{Q_n}{W_n} \tag{9}$$

TABLE 1

PERFORMANCE PARAMETERS AND THEIR CORRESPONDING NORMALIZED PARAMETERS

Design, operating, stack and gas parameters	Normalized parameters
1. Cooling power: Q 2. Acoustic power: W 3. Mean pressure: Pm 4. Sound velocity: u 5. Cross-sectional area of stack: A 6. Operating frequency: f 7. Stack length: L 8. Stack center position: X 9. Dynamic pressure amplitude: Pa	1. Normalized cooling power: Qn = Q / (Pm uA) 2. Normalized acoustic power: Wn = W / (Pm uA) 3. Normalized stack length: Ln = kL 4. Normalized stack center position: Xn = kX 5. Drive ratio: D = Pa/m 6. Normalized temperature difference θn = θ/Tm
10. Temperature difference across stack: θ 11. Mean temperature of gas: Tm	7. Porosity of stack: B = yo/(yo+ l)
12. Half-stack plate spacing: yo 13. Half-stack plate thickness: l 14. Thermal penetration depth: δk	8. Normalized thermal penetration depth: δkn = δk/yo
15. Thermal conductivity of gas: K 16. Isobaric specific heat of gas: Cp 17. Isochoric specific heat of gas: Cv 18. Dynamic viscosity of gas: μ	9. Prandtl number: σ 10. Polytropic coefficient: γ

TABLE 2

DATA USED IN THE PERFORMANCE CALCULATIONS

P_m = 10 bar	u = 1022.4
T_m = 300 K, T_h = 300 K	σ = 0.68
θ_n = 0.25	γ = 1.67
D = 0.02	B = 0.75
f = 400 Hz, k = 2.46 m^{-1}	δ_{kn} = 0.5

4.0 RESULTS AND DISCUSSIONS

4.1 Effect of Stack Length on COPs for Various Centre Positions

The optimum stack positions for the most efficient performance of the thermoacoustic system are found that the most efficient heat transfer occurred at the maximum position (upstream pumping) and the minimum position (downstream pumping) of the temperature change versus position curve at the resonance frequency [9]. The stack performance as a function of the normalized stack length L_n for different normalized stack positions X_n is calculated using Eq. (9). A graph is drawn for COP_s as a function of stack length for the various values of stack center positions as shown in Figure 2. In all the cases, there are maximum and minimum values for stack COP and hence for each stack length, there is an optimal stack position. As can be seen from Figure 2, when the length of the stack increases, the performance peak shifts to larger stack position. The temperature gradient along the stack is proportional to the difference of the gas acoustic pressure near the ends of the stack. This pressure difference is greater when the stack is longer, as the stack is located between node and antinode of the pressure. It is observed that standing wave systems produce acoustic power roughly proportional to the velocity amplitude [2]. This means that the stack needs to be placed near the pressure node of the standing wave. Similarly, the decrease in stack length and center position from the loud speaker increases stack performance. Considering practical reasons and to provide adequate space for instrumentation, we have chosen 80 mm as stack length and center position. This is equivalent to place the hot end of the stack at a distance of 40 mm from the driver.

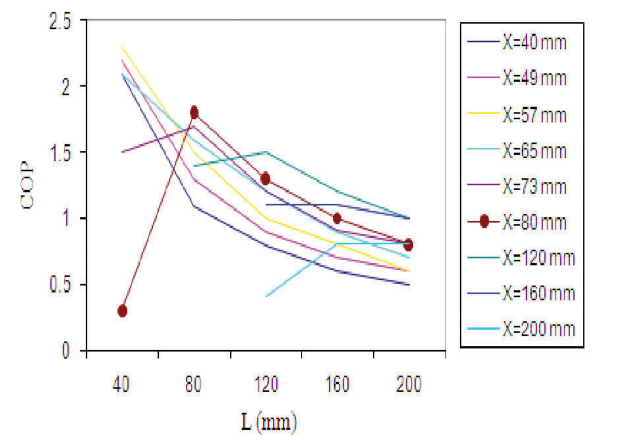

FIG. 2 VARIATION OF STACK COP WITH STACK LENGTHS FOR VARIOUS CENTER POSITIONS

The stack length of 80 mm is very small compared to the wavelength of the acoustic field λ of 2556 mm. Hence, the assumption of short stack is perfectly valid. For the stack center position of 80 mm, the critical temperature gradient is 148.8 K, whereas the actual temperature gradient across the stack end is 75 K, and hence the normalized temperature gradient Γ is found to be 0.5, which is a primary necessity [1] for the device to function as a refrigerator. For a given temperature difference θ, there is a limit for minimum and maximum stack length to behave as a refrigerator. The minimum stack length and is also known as critical stack length L_c which is found to be 40 mm using Eq. (1). The maximum stack length is known by plotting the normalized heat and work fluxes, Q_n and W_n, respectively, as a function of normalized stack length for the normalized stack center position X_n. The two intersection points of the curves representing Q_n and W_n show the minimum and maximum stack length for the designed capacity of the refrigerator and hence the maximum stack length is found to be 160 mm for the optimized stack length of 80 mm, which lies between this limit. Under these conditions, the normalized cooling power, Q_n, is found to be 3.715×10^{-6}.

4.2 Effect of Stack Diameter on the Cooling Power

The cross-sectional area of the stack and hence the stack diameter can be calculated using normalized cooling power equation. The variation of stack diameter for the various values of cooling power is shown in Figure 3. The diameter of the stack depends on the required cooling power and increases with increase in cooling power *Q*.

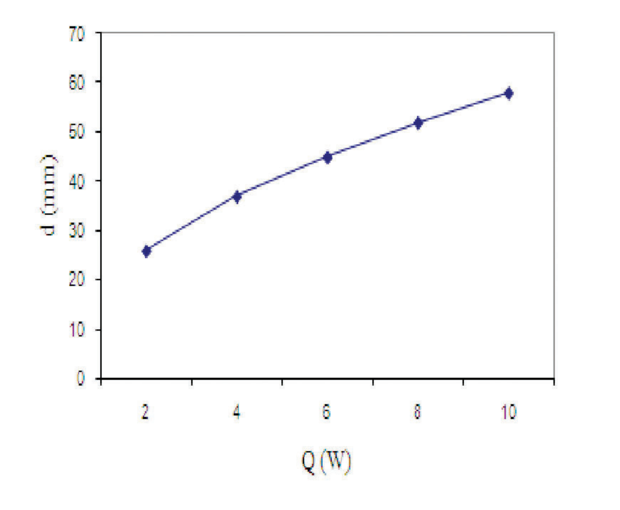

FIG. 3 VARIATION OF STACK DIAMETER WITH COOLING LOADS

4.3 Effect of Stack End Temperature Difference on the Performance

The performance of the stack, COP_S, depends on the temperature difference across the stack and hence the COP_S for the optimized stack length and position of 80 mm each is calculated for the various values of the normalized temperature difference using Eq. (9). The variation in the performance of the stack as a function of temperature difference θ across the stack is shown in Figure. 4. It is observed that the performance of the stack decreases with increase in temperature difference across the stack and is expressed in other words, as the cold-end temperature of the stack decreases, the COP_s also decreases at the constant hot-end temperature of the stack.

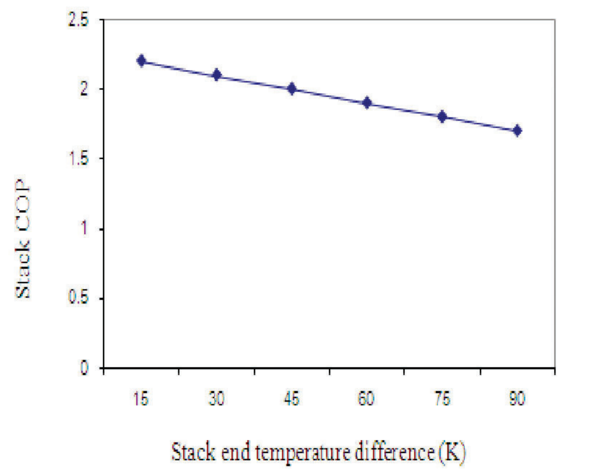

FIG. 4 VARIATION OF COP_S WITH STACK-END TEMPERATURE DIFFERENCES

5.0 CONCLUSIONS

The design procedure for optimizing the stack of a thermoacoustic refrigerator is discussed. For the optimized stack length, the minimum and maximum lengths for the device to behave as a refrigerator are determined. The cross-sectional area and hence the diameter of the stack increases with increase in required cooling power and hence the size. The performance of the stack gradually decreases with increase in temperature difference θ across the stack. The optimum performance of the stack is found to be 1.8 for the design conditions considered, which is 38.5 % higher in comparison with published results [5]. The reasons for improvement are the mean temperature of the gas T_m, sound velocity of gas u and the stack end temperature difference θ. Analysis results show that, higher values for mean temperature of the gas and sound velocity and the lower value for stack-end temperature difference improve the performance of the stack.

REFERENCES

[1] Swift GW. "Thermoacoustic engines", *Acoust Soc J Am.*, Vol. 84, pp. 1146–80, 1988.

[2] Swift GW. "Thermoacoustics–A unifying perspective of some engines and refrigerators", *Acoustical Society of America Publication*, 2002.

[3] Russell DA, Weibull P. Table top thermo acoustic refrigerator for demonstration, *Am. J. Phys*, Vol. 70, No. 12, pp. 1231–1233, 2002.

[4] Tijani MEH. "Loudspeaker-driven thermo-acoustic refrigeration", Ph.D Thesis, Eindhoven University of Technology, 2001.

[5] Tijani MEH, Zeegers JCH, de Waele ATAM. "Design of thermoacoustic refrigerator", Cryogenics, Vol. 42, pp. 49–57, 2002.

[6] Tijani MEH, Zeegers JCH, de Waele ATAM. Construction and Performance of a Thermoacoustic Refrigerator, Cryogenics, Vol. 42, pp. 59–66, 2002.

[7] Tijani MEH, Zeegers JCH, de Waele ATAM, "The optimal stack spacing for thermoacoustic refrigeration", *J. Acoust Soc. Am.*, Vol. 112, No. 1 pp. 128–133, 2002.

[8] Wetzel M, Herman C. "Design optimizat-ion of thermoacoustic refrigerator", *International Journal of Refrigeration*, Vol. 20, pp. 3–21. 1997.

[9] Kim YT and Kim MG. "Optimum positions of a stack in a thermoacoustic heat pump", *J. Kor. Phys. Soc.*, Vol. 36, pp. 279–286, 2000.