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Effective Location of SVC Controller for Small Signal Stability Enhancement in Multi Machine Power Systems

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The problem of small signal stability is usually due to insufficient damping of system oscillations. The power electronic based FACTS devices are effective for system voltage control and power flow control. This paper presents a systematic method of developing the small signal model for stability enhancement in a power system using SVC. While the primary purpose of a Static Var Compensator (SVC) is to regulate bus voltage, it can also improve stability and damping of a power system if located appropriately. This paper proposes a Residue factor to find the location of SVC in multi-machine system. The proposed residue factor was based on the relative participation of the parameters of SVC controller to the critical oscillatory mode. The algorithmic steps for computing the residue factor is proposed, which combined the linearized differential algebraic equation (DAE) model of the power system and the SVC output equations. The effectiveness of the proposed method was demonstrated on the standard WSCC 3 Machine 9-Bus system. All computations were carried out using MATLAB™ v7.6.

Keywords: *Small signal stability, Effective location of SVC, Controllability and Observability residues.*

1.0 INTRODUCTION

As power systems became interconnected, areas of generation were found to be prone to electromechanical oscillations**.** These oscillations have been observed in many power systems worldwide. With increased loading conditions and interconnections the transmission system became weak and inadequate, also load characteristics added to the problem causing spontaneous oscillations. These oscillations may be local to a single generator or a generator plant (local oscillations, 0.8 – 2 Hz), or they may involve different groups of generators widely separated geographically (inter area oscillations, 0.2–0.8 Hz). These uncontrolled electromechanical oscillation may lead to total or partial power interruption [1].

The recent advances in power electronics technology have led to the development of FACTS controllers which are effective candidates for providing secure loading, power flow control and voltage control in transmission systems. These controllers when placed effectively with supplementary stabilizing loops are found to be effective for damping out power system oscillations was discussed in [2].

Presents the basic static Var compensator (SVC) [3] control strategies for enhancing the dynamic and transient stability of a simple two machine system.

The modeling of SVC for transient stability studies were discussed [4]. Proposed a optimal location method for SVC using participation factor analysis [5].

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The dynamic behavior of voltage source converter based FACTS devices for simulation studies was discussed in [6]. These devices were modeled as current injections for dynamic analysis.

This paper proposes a Residue factor to find the location of SVC in multi-machine system. The proposed residue factor was based on the relative participation of the parameters of SVC controller to the critical mode was discussed [2]. The electrical circuit dynamics of the synchronous machines are modeled using the standard two axis model [8]. The following section presents the mathematical modeling details of the FACTS devices enhancement of dynamic stability.

This paper also provides a generalized method of developing small signal model of power system with shunt connected FACTS devices. The electrical circuit dynamics of the synchronous machines are modeled using the standard two axis model. This paper also proposes an optimal location method for maximizing the damping ratio of the swing mode in the power system.

The following section presents the mathematical modeling details of the FACTS devices and the optimal tuning procedure for enhancement of dynamic stability.

2.0 MATHEMATICAL MODELING

The linearized state equations in per unit form are given below [8].

$$\Delta \dot{E}'_{di} = \frac{1}{\tau'_{qoi}}\left(-\Delta E'_{di} - \left(x_{qi} - x'_{qi}\right)\Delta I_{qi}\right)$$

$$\Delta \dot{E}'_{qi} = \frac{1}{\tau'_{doi}}\left(\Delta E_{FDi} - \Delta E'_{qi} + \left(x_{di} - x'_{di}\right)\Delta I_{di}\right) \quad ...(1)$$

$$\Delta \dot{\omega}_i = \frac{1}{\tau_{ji}}\{\Delta T_{mi} - D\Delta\omega_i - \Delta T_{ei})$$

$$\Delta \dot{\delta}_i = \Delta\omega_i$$

$$i = 1, 2 - - n$$

where the state variables are

E'_d - direct axis component of voltage behind transient reactance.

E'_q- quadrature axis component of voltage behind transient reactance.

ω - Angular velocity of rotor

δ – Rotor angle.

2.1 Modeling of shunt FACTS controller

For the purpose of developing the small signal stability program all the series connected FACTS devices are represented as current injections in two nodes of the network [7]. However, if the device is a shunt connected device then the injections are confined only to one node (Figure 1).

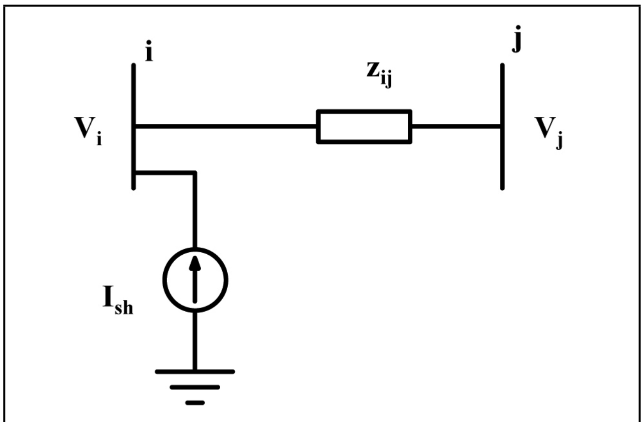

FIG. 1 CURRENT INJECTION MODEL OF SVC.

The change in bus voltage (ΔV_S) due to shunt connected FACTS device in the network is expressed in terms of the state variables from the last row of the matrix equation given by

$$\begin{bmatrix}\Delta\bar{I}_1\\ \Delta\bar{I}_2\\ \Delta\bar{I}_{sh}\end{bmatrix} = \begin{pmatrix} Y_{11}e^{j\theta_{11}} & Y_{12}e^{j(\theta_{12}-\delta_{120})} & Y_{13}e^{j(\theta_{13}-\delta_{130})}\\ Y_{21}e^{j(\theta_{21}-\delta_{210})} & Y_{22}e^{j\theta_{22}} & Y_{23}e^{j(\theta_{23}-\delta_{230})}\\ Y_{31}e^{j(\theta_{31}-\delta_{310})} & Y_{32}e^{j(\theta_{32}-\delta_{320})} & Y_{33}e^{j\theta_{33}} \end{pmatrix}\begin{bmatrix}\Delta\bar{E}_1\\ \Delta E_2\\ \Delta\bar{V}_S\end{bmatrix}$$

$$-j\sum_{k=1}^{n}\begin{bmatrix}\bar{V}_{k0}Y_{1k}e^{j(\theta_{1k}-\delta_{1k0})}\Delta\delta_{1k}\\ \bar{V}_{k0}Y_{2k}e^{j(\theta_{2k}-\delta_{2k0})}\Delta\delta_{2k}\\ \bar{V}_{k0}Y_{3k}e^{j(\theta_{3k}-\delta_{3k0})}\Delta\delta_{3k}\end{bmatrix} \quad ....(2)$$

2.2 SVC Modeling

The SVC dynamic model used for linear analysis is shown in Figure 2. With an additional stabilizing signal, supplementary control superimposed on the voltage control loop of an SVC can provide damping of system oscillations [3].

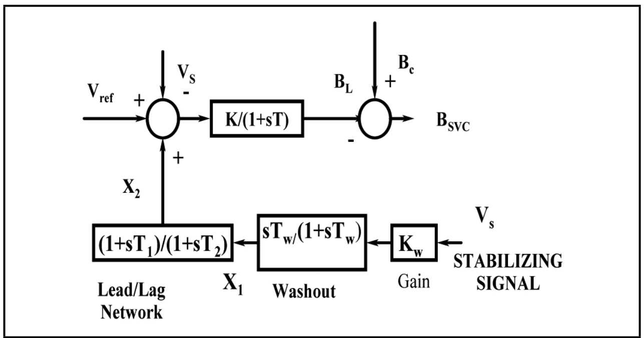

FIG. 2 DYNAMIC MODEL OF SVC.

From the matrix equation (2), the change in network current with the introduction of SVC in the d-q reference frame is given below

$$
\begin{aligned}
\Delta I_{qi} &= G_{ii}\Delta E'_{qi} - B_{ii}\Delta E'_{di} \\
&+ \sum_{k\neq i,j}[(G_{ki}\cos\delta_{kio} - B_{ki}\sin\delta_{kio})\Delta E'_{qk} \\
&-(B_{ki}\cos\delta_{kio} + G_{ki}\sin\delta_{kio})\Delta E'_{dk} \\
&-(G_{ki}\sin\delta_{kio} + B_{ki}\cos\delta_{kio})\Delta\delta_{ki}E'_{qko} \\
&+(B_{ki}\sin\delta_{kio} - G_{ki}\cos\delta_{kio})\Delta\delta_{ki}E'_{dko}] \quad \ldots (3) \\
&+\sum_{j}[(G_{ji}\cos\delta_{jio} - B_{ji}\sin\delta_{jio})\Delta V_{srj} \\
&-(B_{ji}\cos\delta_{jio} + G_{ji}\sin\delta_{jio})\Delta V_{smj} \\
&-(G_{ji}\sin\delta_{jio} + B_{ji}\cos\delta_{jio})\Delta\delta_{ji}V_{srjo} \\
&+(B_{ji}\sin\delta_{jio} - G_{ji}\cos\delta_{jio})\Delta\delta_{ji}V_{smjo}]
\end{aligned}
$$

$$
\begin{aligned}
\Delta I_{di} &= B_{ii}\Delta E'_{qi} + G_{ii}\Delta E'_{di} \\
&+ \sum_{k\neq i,j}[(G_{ki}\sin\delta_{kio} + B_{ki}\cos\delta_{kio})\Delta E'_{qk} \\
&+(G_{ki}\cos\delta_{kio} - B_{ki}\sin\delta_{kio})\Delta E'_{dk} \\
&+(G_{ki}\cos\delta_{kio} - B_{ki}\sin\delta_{kio})\Delta\delta_{ki}E'_{qko} \\
&-(B_{ki}\cos\delta_{kio} + G_{ki}\sin\delta_{kio})\Delta\delta_{ki}E'_{dko}] \quad \ldots (4) \\
&+\sum_{j}[(G_{ji}\sin\delta_{jio} + B_{ji}\cos\delta_{jio})\Delta V_{srj} \\
&+(G_{ji}\cos\delta_{jio} - B_{ji}\sin\delta_{jio})\Delta V_{smj} \\
&+(G_{ji}\cos\delta_{jio} - B_{ji}\sin\delta_{jio})\Delta\delta_{ji}V_{srjo} \\
&-(B_{ji}\cos\delta_{jio} + G_{ji}\sin\delta_{jio})\Delta\delta_{ji}V_{smjo}]
\end{aligned}
$$

The linearized state equations of the SVC for small signal analysis is given below

$$
\begin{aligned}
\Delta\dot{B}_L &= \frac{K}{T}\Delta X_L + \frac{K}{T}\left[\left(\frac{T_1}{T_2}\right)K_W - 1\right]\Delta V_S \\
&- \frac{\Delta B_L}{T} - \frac{K}{T}\left(\frac{T_1}{T_2}\right)\Delta X_W \qquad \ldots (5)
\end{aligned}
$$

The differential equations connected with the washout and lead lag filter are [9]

$$\Delta\dot{X}_W = \frac{K_W}{T_W}\Delta V_S - \frac{\Delta X_W}{T_W} \qquad \ldots (6)$$

$$\Delta\dot{X}_L = \frac{1}{T_2}\left(1 - \frac{T_1}{T_2}\right)(K_W\Delta V_S - \Delta X_W) - \frac{\Delta X_L}{T_2} \qquad \ldots (7)$$

Substituting (3) and (4) in the differential of the synchronous machine (1) and SVC dynamic equations (5–7) yields the system state space matrix.

The complete set of state variables describing the dynamics of the synchronous machine with the inclusion of the SVC in the network is as follows.

$$x^T = [E'_d, E'_q, \omega, \delta, B_L, X_W, X_L] \qquad \ldots (8)$$

3.0 ALGORITHM FOR FINDING THE EFFECTIVE LOCATION

The critical steps for the small signal stability evaluation in multi-machine power systems with FACTS devices are listed below

- **Step 1:**
  Get the transmission line data, bus data and generator data for the given system and form the bus admittance matrix from the given transmission line data.
- **Step 2:**
  Eliminate all the nodes except for the internal generator nodes and FACTS connected nodes. For shunt connected devices, there is

only one node whereas for series connected devices, there are two nodes.

- **Step 3:**
  Obtain the Y_{red} matrix from the reduced network,

$$Y_{red} = Y_{nn} - (Y_{nr} \times (Y_{rr})^{-1} \times Y_{rn}) \quad \dots (9)$$

- **Step 4:**
  For the formation of state space model, the initial conditions are computed in advance. ($E`_{qo}$, $E`_{do}$, $I`_{qo}$, $I`_{do}$).

- **Step 5:**
  Formulate the differential equations for pE'_q, pE'_d, $p\delta$, $p\omega$ with additional state variables due to FACTS devices as $\dot{\underline{x}} = [\underline{A}]\,\underline{x}$ after eliminating the algebraic equations.

- **Step 6:**
  From the state space matrix, the eigen values are to be calculated and damping ratio are calculated for the electromechanical modes ($\lambda = -\sigma \pm j\omega$) as

$$\varsigma = -\sigma / (\sqrt{\sigma^2 + \omega^2} \quad \dots (10)$$

- **Step 7:**
  Compute the participation matrix from the right and left eigen vectors of the A-matrix.

3.0 SMALL SIGNAL STABILITY ENHANCEMENT AND DETERMINATION OF EFFECTIVE LOCATION

The test system considered for small signal stability investigation is the 3 Machine 9 Bus system [8] (Figure 3). The 3 machine test system is operating with the load at bus 5 carrying 125 MW, bus 6 carrying 90 MW and bus 8 supplying 100 MW. The real power generations are 71.3, 163 and 85 in generators 1, 2 and 3 respectively.

Eigen value analysis results of the 3 Machine 9 Bus system around the operating state mentioned above is displayed in Table 1.For verification of results Machine 1 is considered as classical model and Machines 2, 3 as two axis models.

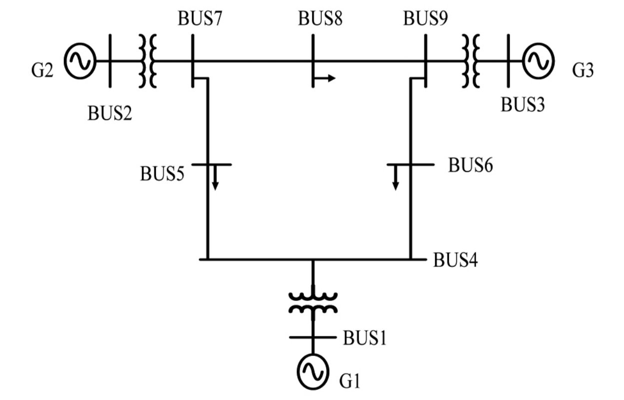

FIG. 3 3 MACHINE 9-BUS SYSTEM.

TABLE 1
EIGEN VALUE ANALYSIS FOR 3 MACHINE 9-BUS SYSTEM

Eigen values	Damping ratio	Associated states
–0.0026640944118 ± 0.034642495927i	0.07345	δ_{13}, ω_3
–0.0006213675132 ± 0.02297764868i	0.02654	δ_{12}, ω_2

It can be observed that damping ratios of the swing modes (local modes) are poor (0.07345 and 0.02654). The eigen value calculations match with the results given in the ref. [8].

Table 2 displays the effect of FACTS stabilizer on the dynamic stability of the 3 machine 9 bus system. For this analysis all the machines are modeled using the two axis model, to accurately model the small signal behavior of the system. The shunt connected FACTS device (SVC) is located at load buses. The data for the FACTS stabilizers are listed in the appendix.

TABLE 2
EIGEN VALUE ANALYSIS –EFFECT OF FACTS STABILIZERS

Without damping controller	With SVC (atbus-5)	With SVC (at bus-6)	With SVC (at bus-8)
–0.00243 ± 0.03444i ζ = 0.07345	–0.00232 ± 0.02456i ζ=0.09432	–0.00229 ± 0.02244i ζ=0.10188	–0.00167 ± 0.01548i ζ=0.10751

–0.00053 ± 0.02281i ζ=0.0265	–0.00060 ± 0.01058i ζ=0.0575	–0.00063 ± 0.01197i ζ=0.0531	–0.00099 ± 0.01155i ζ=0.0854

From the table it can be observed that with SVC in the network the damping ratio of the modes improve when it is located at bus-8.

4.0 RESIDUES

Let us start from the mathematical model a dynamic system expressed in terms of a system of nonlinear differential equations:

$$\dot{x} = F(x, t) \qquad \text{.... (11)}$$

If this system of non-linear differential equations is linearized around an operating point of interest $x = x_0$, it results in:

$$\Delta\dot{x} = A\Delta x(t) \qquad \text{.... (12)}$$

Assume that an input u(t) and an output y(t) of the linear dynamic system (12) have defined:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \qquad \text{... (13)}$$

Considering (13) with single input and single output (SISO) and assuming D = 0, the open loop transfer function of the system can be obtained by:

$$\begin{aligned} G(s) &= \frac{y(s)}{u(s)} \\ &= C(sI\text{-}A)^{-1}B \end{aligned} \qquad \text{.... (14)}$$

The transfer function G(s) can be expanded in partial fractions of the Laplace transform of y in terms of C and B matrices and the right and left eigen vectors as:

$$G(s) = \sum_{i=1}^{N} \frac{C\phi_i \psi_i B}{(s-\lambda_i)} = \sum_{i=1}^{N} \frac{R_{ijk}}{(s-\lambda_i)} \qquad \text{.... (15)}$$

where,

R_{ijk} is the residue associated with i^{th} mode, j^{th} output and k^{th} input. R_{ijk} can be expressed as:

$$R_{ijk} = C_j v_i w_i B_k \qquad \text{.... (16)}$$

v_i and w_i denote the right and left eigen vectors associated with the i^{th} critical electromechanical mode. This can be expressed in terms of mode controllability and observability. The controllability of mode i from the k^{th} input is given by

$$CI_{ik} = \left| w_i B_k \right| \qquad \text{.... (17)}$$

The measure of mode observability of mode i from output j is given by

$$Obsv_{ij} = \left| C_j v_i \right| \qquad \text{.... (18)}$$

It is clear that:

$$\left| R_{ijk} \right| = \left| C_j v_i w_i B_k \right| = obsv_{ij} \times cont_{ik} \qquad \text{... (19)}$$

Each term in the denominator, R_{ijk}, of the summation is a scalar called residue. The residue R_{ijk} of a particular critical electromechanical mode i gives the measure of that mode's sensitivity to a feedback between the output y and the input u; it is the product of the mode's observability and controllability.

For that critical electromechanical mode of the interest, residues at all locations have to be calculated. The largest residue then indicates the most effective location of FACTS device.

5.0 RESULTS

The effectiveness of the proposed method was tested on WSCC 3 machine, 9-bus system. The results for the system are presented as follows:

The system consists of 3 generators, three fixed admittance loads and 6 branches with generator 1 taken as reference generator. The equivalent power system of WSCC 3 machine 9 bus system is depicted in Figure 3.

It is observed from Table 3 that load bus 8 has the maximum residue factor value. Thus, bus 8 is the most effective location for placement of SVC device. The eigen value analysis results computed were also verified using the linear simulations in the industry standard EUROSTAG™ software package.

TABLE 3

RESIDUE FACTOR AT DIFFERENT LOAD BUSES

Location of SVC bus	Residue factor
5	0.02394
6	0.03143
8	0.08916

6.0 CONCLUSION

This paper presented the mathematical model for locating a shunt connected FACTS device for small signal stability enhancement in a multi-machine power system. Much of the earlier work relevant to small signal stability enhancement using FACTS stabilizers has used the classical model of the synchronous machine neglecting the effect of electrical circuit dynamics. This paper has presented a systematic and generalized approach for small signal modeling and a method called 'Location index' for the location of FACTS device. It should be noted that this paper makes use of local feedbacks as stabilizing signals for the location of FACTS based damping controllers.

APPENDIX

The data for FACTS devices are given below in p.u.

R	:	0
X	:	0.025
R_C	:	0.077
C	:	0.2592
K	:	10
T_1	:	1.1
K_d	:	10
T_2	:	0.05
K_{Mac}	:	1
T_{Mac}	:	0.01
K_p	:	10
K_I	:	1
K_{Mdc}	:	10
T_{Mdc}	:	0.01

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