The Journal of CPRI,
Vol. 9, No. 1, March 2013 pp. 89–96

Sub-synchronous Resonance through Torsional Mode Interactions: An Analysis

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Interaction between electrical and mechanical systems, generally termed as sub-synchronous resonance (SSR) in power system, is not a straightforward problem to analyze as it involves detailed modeling of both systems. From the literature it is found that in most of the cases the SSR analysis is demonstrated through the IEEE first-benchmark system which contains many intricate modal interactions. To motivate the students to take up such standard systems for analysis, in this paper an example is presented which brings out the issues associated with the analysis by employing simplified models for generator and mechanical systems. Model and design equations are systematically derived so that any variations can be tried with ease. Case studies are presented to demonstrate the concept of modal interactions by studying the systems both in isolation and in combination. Eigenvalue analysis is carried out in addition to time-domain simulation to show that SSR is purely a discrete event.

Keywords: *Series compensation, Sub-synchronous resonance and FFT.*

1.0 INTRODUCTION

In a stability constrained power system, where transmission system expansion is restricted due to limited right-of-way, compensation using Fixed Series Capacitors (FSC) has been the natural choice for improving the power transfer capability of transmission systems [1]. However, in such systems there is a possibility of adverse interaction between the generator-turbine mechanical system and the electrical network compensated with fixed series capacitors - known as Sub synchronous Resonance (SSR) [2–6]. From the literature [7–12] it is clear that there has been a continued effort to understand the SSR phenomenon employing various techniques such as Eigen value analysis, frequency scanning and time-domain techniques and to investigate different countermeasures for mitigating the SSR effects. Extensive studies both on computer simulations and field investigations [13–16] have shown that analysis of SSR is not straight forward since it requires detailed modelling of components. Further, in most of these studies the IEEE First-Benchmark System (FBS) [17] has been employed which involves interactions of many modes. With such a system it is not easy to demonstrate or understand the concept of modal-interactions between the electrical and mechanical systems.

In an attempt to illustrate this complex phenomenon an example has been developed in this paper. In this example a simplified generator model has been employed which acts as a media for exchange of energy between electrical and mechanical systems. The series-capacitor compensated transmission line is modeled as a series RLC circuit in a usual manner showing its natural frequency depending on the level of compensation. The mechanical system is designed in such a way that it possesses only one torsional mode. The modal equations are derived to facilitate easy choice of modal frequency.

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Each system is studied individually to clearly bring out their natural behavior and finally the systems are interconnected to demonstrate the mutual excitation of the systems depending on the level of compensation of the transmission system. It is felt that such an example not only introduces the difficult concept of SSR in powers system, but also motivates the graduate power engineering students to learn the more complicated IEEE FBS.

2.0 ANALYSIS OF SSR

The one line diagram of the system is shown in Figure 1. Here, a very simplified model has been employed for the generator electrical circuit. In the figure = $\overline{V}_g < \delta_g$ denotes the generator voltage where V_g is held constant at the specified value and only δ_g is assumed to change as constrained by the mechanical system. The mechanical system is designed to have a single natural (torsional) frequency.

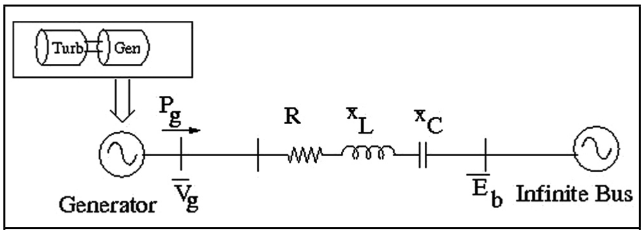

FIG. 1 A SIMPLE POWER SYSTEM MODEL FOR SSR ANALYSIS.

2.1 Mechanical System

The mechanical system which consists of two rotor masses corresponding to the turbine and the generator is represented by a two mass-spring-dashpot system as shown in Figure 2.

Using the standard notations [6], the equations of the two rotor-mass systems are summarized as follows:

Turbine:

$$\frac{d\delta_t}{dt} = \left(\omega_t - \frac{\omega_0}{\omega_B}\right)\omega_B$$

$$2H_t\frac{d\omega_t}{dt} = \begin{bmatrix} T_{mt} - K_{tg}\left(\delta_t - \delta_g\right) \\ -D_{tg}\left(\omega_t - \omega_g\right) - D_t\omega_t \end{bmatrix} \quad \text{... (1)}$$

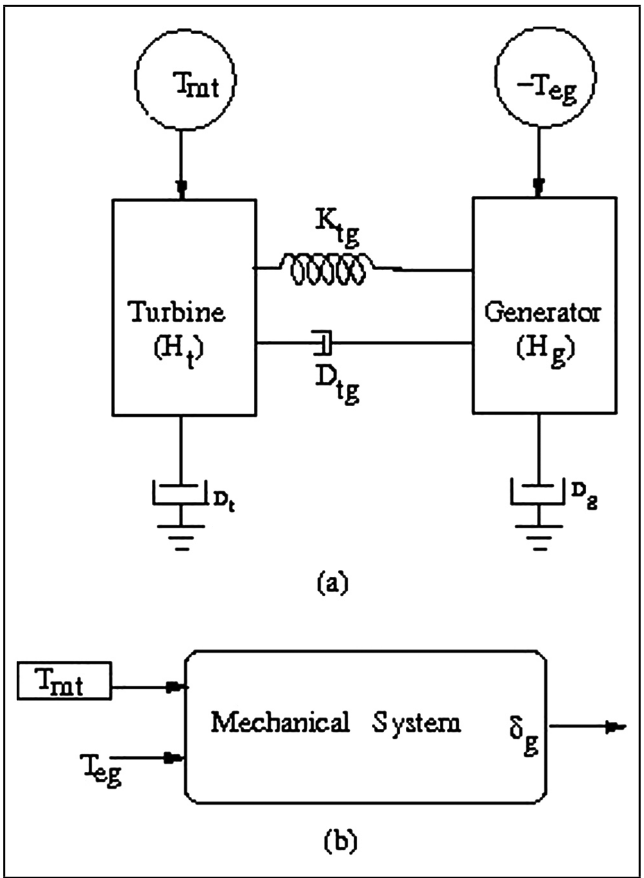

FIG. 2 MECHANICAL MODEL OF A ROTOR-SHAFT TORSIONAL SYSTEM WITH TWO MASSES.

Generator:

$$\frac{d\delta_g}{dt} = \left(\omega_g - \frac{\omega_0}{\omega_B}\right)\omega_B$$

$$2H_g\frac{d\omega_g}{dt} = \begin{bmatrix} -T_{eg} - K_{tg}\left(\delta_g - \delta_t\right) \\ -D_{tg}\left(\omega_g - \omega_t\right) - D_g\omega_g \end{bmatrix} \quad \text{... (2)}$$

Writing the above equations only in terms of angles, i.e. $\underline{\delta} = [\delta_t\ \delta_g]^T$, and finally in the matrix form, we get,

$$[M]\, p^2\underline{\delta} + [D]\, p\underline{\delta} + [K]\underline{\delta} = \underline{T}_m - \underline{T}_e = \underline{T} \quad \text{.... (3)}$$

Where,

$$[M]=\begin{bmatrix} M_t & 0 \\ 0 & M_g \end{bmatrix}$$

$$[K]=\begin{bmatrix} K_{tg} & -K_{tg} \\ -K_{tg} & K_{tg} \end{bmatrix} [D']=\begin{bmatrix} \dfrac{D_{tg}+D_t}{\omega_B} & -\dfrac{D_{tg}}{\omega_B} \\ -\dfrac{D_{tg}}{\omega_B} & \dfrac{D_{tg}+D_g}{\omega_B} \end{bmatrix}$$

$$\underline{T}_m=\begin{bmatrix} T_t - D_t\left(\dfrac{\omega_0}{\omega_B}\right) \\ 0 \end{bmatrix} \underline{T}_e=\begin{bmatrix} 0 \\ T_{eg}+D_g\left(\dfrac{\omega_0}{\omega_B}\right) \end{bmatrix}$$

With $M_t=\dfrac{2H_t}{\omega_B}$ and $M_g=\dfrac{2H_g}{\omega_B}$

To determine the modal (natural) frequency of the spring-mass system, a modal spring-mass model is developed employing the procedure suggested in [6]. A transformation matrix [Q] is obtained as the matrix of right eigen vectors of the matrix $[M]^{-1}[K]$, with each vector of [Q] be selected such that its element corresponding to the generator rotor is unity. Using this matrix, modal matrices are obtained such that they are diagonal. We have,

$[M]^m= [Q]^T[M][Q]$ and $[K]^m=[Q]^T[K][Q]$

Where we have

$$[Q]=\begin{bmatrix} 1 & -\dfrac{M_g}{M_t} \\ 1 & 1 \end{bmatrix}$$

$$[M]^m=\begin{bmatrix} (M_t+M_g) & 0 \\ 0 & \dfrac{M_g(M_t+M_g)}{M_t} \end{bmatrix}$$

$$[K]^m=\begin{bmatrix} 0 & 0 \\ 0 & \dfrac{K_{tg}(M_t+M_g)^2}{M_t^2} \end{bmatrix}$$

The radian frequency of the torsional mode is given by,

$$\omega_i=\sqrt{\frac{K_i^m}{M_i^m}} \qquad \text{.... (4)}$$

Where, K_i^m and M_i^m denote i^{th} diagonal elements of $[K^m]$ and $[M^m]$ respectively. Therefore, the natural frequency of torsional mode of oscillation, f_m, is given by

$$f_m=\frac{1}{2\pi}\sqrt{\frac{K_{tg}}{\left(\dfrac{M_tM_g}{M_t+M_g}\right)}} \qquad \text{.... (5)}$$

2.2 Electrical System

The equivalent circuit for the electrical network of Figure 1 is shown in Figure 3. In the figure R and x_L denotes the resistance and total inductive reactance of the circuit (including the generator), respectively. x_C denotes the series capacitive reactance of the FSC.

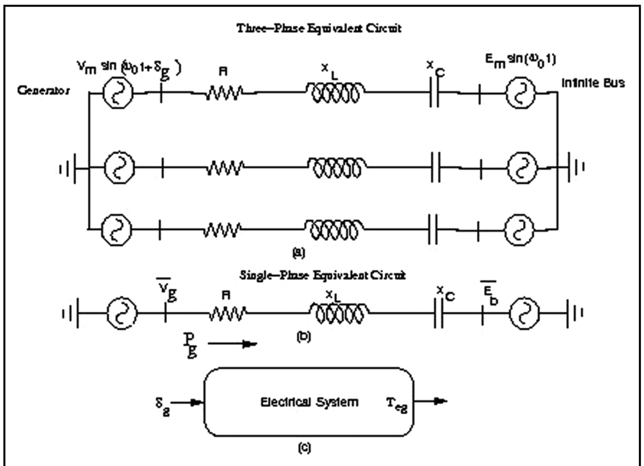

FIG. 3 SERIES CAPACITOR COMPENSATED ELECTRICAL NETWORK.

The resonance frequency, f_{er}, of a series compensated transmission line is given by

$$f_{er}=f_0\sqrt{\frac{x_C}{x_L}} \qquad \text{.... (6)}$$

Where, f_0 is the nominal system frequency.

Since normally $x_C < x_L$, this implies that $f_{er} < f_0$, signifying that resonance phenomenon occurs at a frequency below the nominal frequency.

$$L\frac{di_a}{dt} = v_a - Ri_a - v_{Ca} - E_{ba} \quad \text{.... (7)}$$

$$C\frac{dv_{Ca}}{dt} = i_a \quad \text{.... (8)}$$

The differential equation for the electrical system for *a*-phase is written as

$$\begin{aligned} V_a &= V_m \sin\sin(\omega_0 t + \delta_g) \\ E_{ba} &= E_m \sin\sin(\omega_0 t) \end{aligned} \quad \text{.... (9)}$$

Where

$$V_m = \sqrt{\frac{2}{3}}V_g \quad \text{and} \quad E_m = \sqrt{\frac{2}{3}}E_b$$

The above equations are similarly written for other two phases.

2.3 Case Studies

Case studies have been carried out with the following system parameters in per unit.

Mechanical system: K_{tg} = 41.88, D_{tg} = 0.8, D_t = 0.6, D_g = 0.9, H_t=0.72727, H_g=1.6.

Electrical network: R=0.002, x_L=0.5 and $x_C = k_c x_L$ with k_c= degree of compensation, f_0=60 Hz.

The system is operated with V_g=1.0, P_g=1.0,

$\bar{E}b$=1.0<0° - see Fig. 3. Neglecting mechanical damping and using (5) the torsional frequency is given by f_m=20 Hz or 125.66 rad/s.

The SSR analysis has been carried in the following steps employing two values for k_c:

1. Only electrical system.
2. Electrical system is interfaced to mechanical system *without* connecting mechanical variable to the electrical system in turn - partial interfacing.
3. Interfacing electrical to mechanical system *with* mechanical variable connected to electrical system - complete interfacing.

2.3.1 Only Electrical System

To start with, the electrical system alone, see Figure 3, is analyzed using (7) and (8). The magnitude of the terminal voltage, V_g is assumed to be constant at the specified value. Note that δ_g is held fixed at the nominal value implying that the mechanical system dynamics are not considered. For k_c= 0.45 and using (6), we get the resonance frequency of the electrical system as f_{er} = 40.25 Hz. Without any disturbance the generator torque, T_{eg}, is constant as shown in Figure 4. This corresponds to only power frequency component in generator currents.

Now, the system is perturbed by 0.01 p.u. step reduction in the infinite bus voltage, initiated at t = 0.5s and lasting for 0.01s. This makes the generator current to possess a frequency component, f_{er} = 40.25 Hz in addition to the power frequency. This leads to an oscillatory component of frequency, $(f_0 – f_{er})$ = 19.75 Hz, which is referred to as sub synchronous frequency, in torque. This is clearly seen in the figure with disturbance. It is to be noted that the generator torque is calculated as $T_{eg}=v_a i_a+v_b i_b+v_c i_c$.

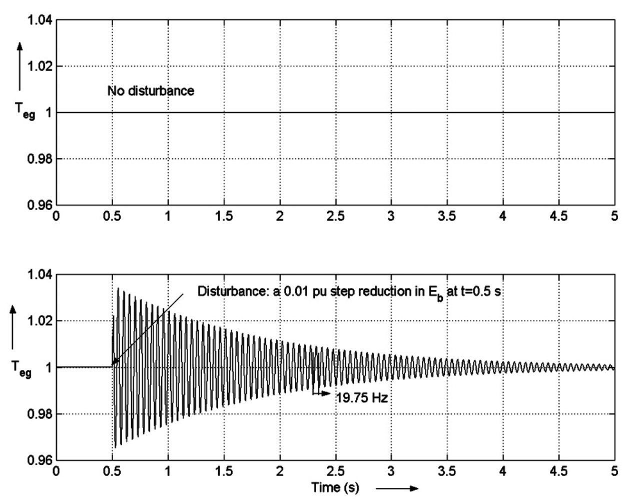

FIG. 4 T_{EG} PLOT WITH AND WITHOUT DISTURBANCE, $X_C = 0.45\ X_L$.

2.3.2 Partial Interfacing

In this case the electrical system is interfaced to mechanical system without interfacing the mechanical variable δ_g to the electrical system. This has been achieved with a configuration as shown in Figure 5. The mechanical system is implemented using (1) and (2).

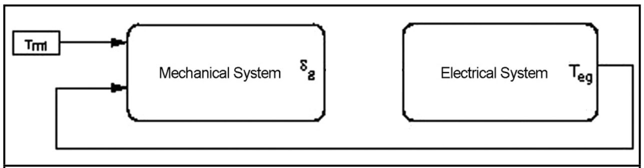

FIG. 5 CONFIGURATION TO STUDY EXCITATION OF TORSIONAL MODE: PARTIAL INTERACTION

In this system the following case studies are carried out:

Case-A: Here, for k_c= 0.45, when the system is in steady-state, the electrical system is perturbed as mentioned above. Since the torque contains a component of frequency, $(f_0\text{-}f_{er})$ = 19.75 Hz, which is close to the torsional frequency, f_m=20 Hz, the torsional mode is excited - see Figure 6. However, note that the oscillations are not sustained as the interfacing is only in one way.

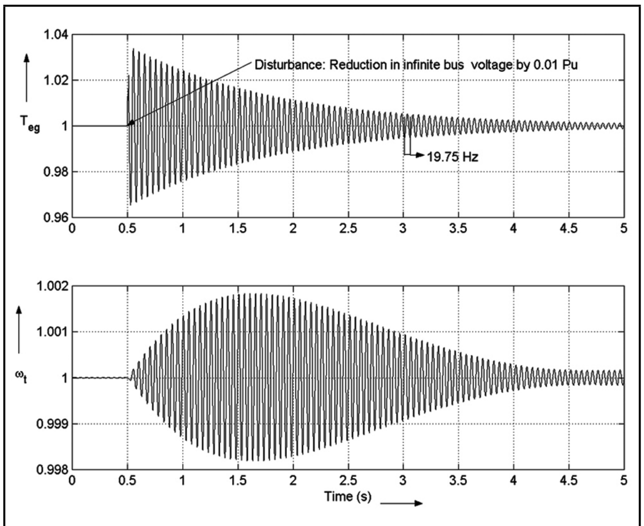

FIG. 6 EXCITATION OF TORSIONAL MODE: PARTIAL INTERACTION FOR X_C = 0.45 X_L.

Case-B: The above case study has been repeated with k_c= 0.25, which results in f_{er}= 30 Hz. Since frequency of the sub synchronous component in torque is $(f_0–f_{er})$ = 30 Hz, which is not in the vicinity of the torsional frequency, f_m=20 Hz, the torsional mode does not get excited. This can be inferred from ω_t plot in Figure 7.

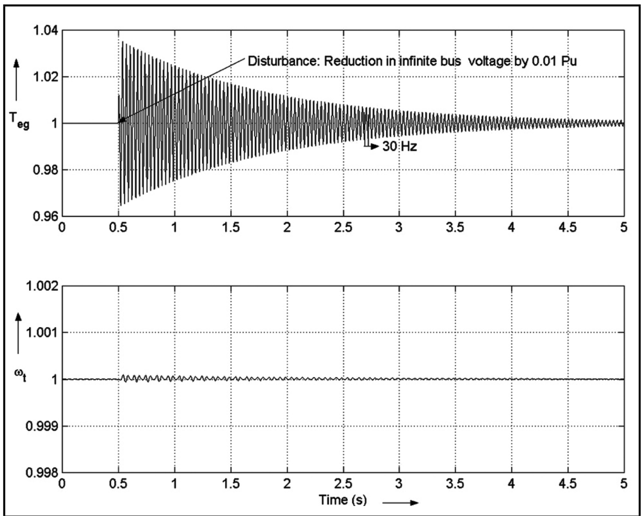

FIG. 7 EXCITATION OF TORSIONAL MODE: PARTIAL INTERACTION FOR X_C = 0.25 X_L

2.3.2 Complete Interfacing

Having considered the partial interaction among the systems, in order to study the SSR phenomenon, the block diagram shown in Figure 8 is considered in which the mechanical variable δ_g obtained from (2) is used in (9) of electrical system.

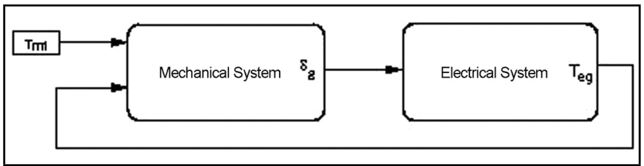

FIG. 8 COMPLETE BLOCK DIAGRAM FOR SSR STUDY.

In this system the following tests are considered:

Case-I: With the system configuration as in Figure 8, the parameters selected in Case-A are adopted. This excites the torsional mode and the rotor exhibits sustained oscillations at f_m=20 Hz since now the interaction is both ways. The resulting oscillations in δ_g variable modulates the generator voltage at two dominant frequencies, one at $(f_0–f_m)$ = 40 Hz and another at (f_0+f_m) = 80 Hz [6]. This has been demonstrated in Figure 9 by extracting the frequency components in v_a.

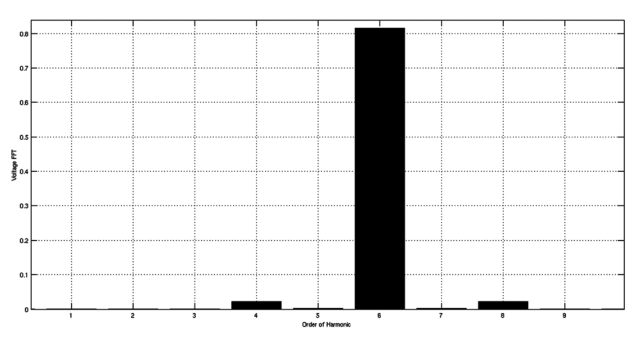

FIG. 9 FFT OF V_A WAVEFORM FOR $X_C = 0.45\ X_L$ WITH 10 Hz AS THE BASE FOR FFT.

Since the sub synchronous frequency component in voltage is close to f_{er} (= 40.25 Hz), it leads to a large sub synchronous currents in i_a - see Figure 10. From the figure it can be seen that beyond 0.5s the i_a waveform possesses other frequency components in addition to power frequency. This fact is further substantiated by determining the RMS value of the line current. Until 0.5s a constant RMS line current denotes only power frequency component in i_a and beyond 0.5s the RMS current contains a dominant frequency (f_0-f_{er}) = 19.75 Hz. This in turn produces sub synchronous torque component which will reinforce the rotor oscillations at frequency, f_m. Such a cumulative process causes the coupled electromechanical system to experience oscillations of large magnitudes - see Figure 11.

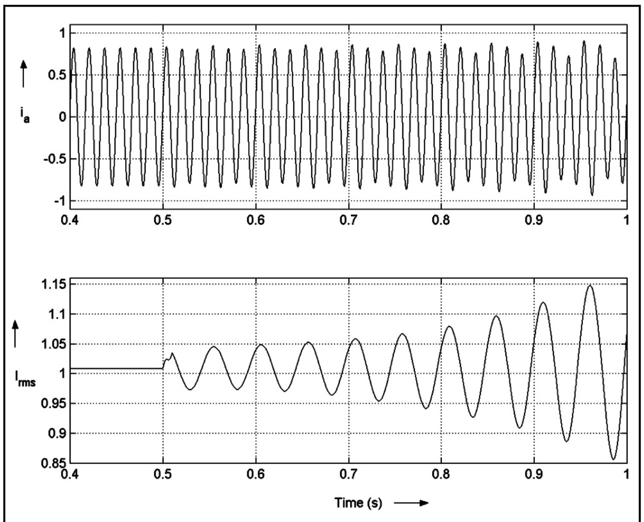

FIG. 10 I_A AND I_{RMS} WAVEFORMS FOR $X_C = 0.45\ X_L$.

In order to support the stability information obtained from the time-domain simulation, an Eigen value analysis is carried out by line-arising the equations in synchronous reference frame [6]. The state vector is chosen as $\underline{x} = [v_{cD}, v_{cQ}, i_D, i_Q, \delta_g, \delta_t, \omega_g, \omega_t]^T$. The results are tabulated in Table 1. From the table, it is seen that as the network sub-synchronous frequency is close to the torsional mode, the torsional mode gets destabilized due to interaction.

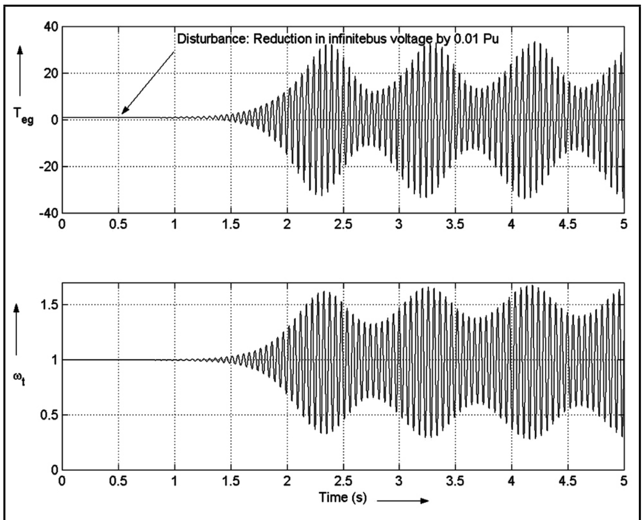

FIG. 11 TORSIONAL INTERACTION STUDY FOR $X_{C = 0.45}\ X_L$

TABLE 1

EIGENVALUES FOR CASE-I, $x_C = 0.45\ x_L$

Eigen values	Comments
–0.7539 ± j 629.83	Super synchronous mode
–5.8357 ± j 124.52	Sub-synchronous mode
4.4840 ± j 124.54	Torsional mode
–0.1492 ± j 16.912	Swing mode

Case-II: In this case, the parameters selected in Case-B are adopted. The corresponding Eigen values are tabulated in Table 2. Here, since frequency of the sub synchronous torque component, 187.7/(2π) = 29.87 Hz, is not in

TABLE 2

EIGENVALUES FOR CASE-I, $x_C = 0.25\ x_L$

Eigen values	Comments
–0.7538 ± j 565.41	Super synchronous mode
–0.7615 ± j 187.70	Sub-synchronous mode
–0.5821 ± j 126.28	Torsional mode
–0.1573 ± j 14.157	Swing mode

the vicinity of the torsional frequency, 126.28/(2π) = 20.09 Hz, the torsional interaction is not sustained. Therefore, the system remains stable as predicted by the Eigen values. These predictions are validated by the time-domain simulation as shown in Figure 12.

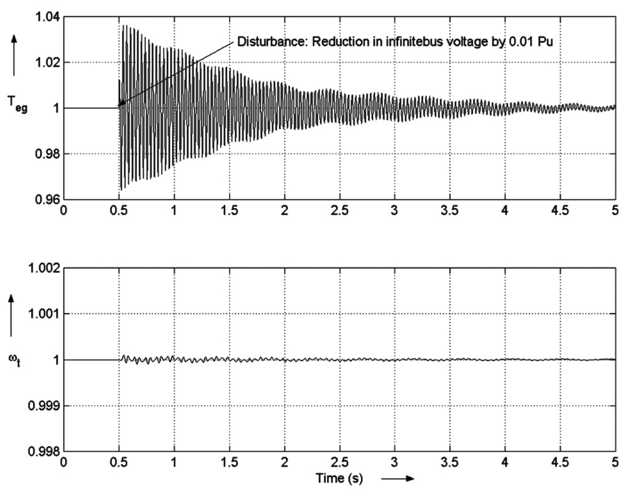

FIG. 12 TORSIONAL INTERACTION STUDY FOR $X_C = 0.25\ X_L$

The above tutorial example illustrates the following important concepts:

- Occurrence of SSR in an electromechanical system denotes a state where electrical systems exchange energy with the mechanical systems through the synchronously revolving rotor systems.
- Torsional frequencies are generally in the sub-synchronous frequency range. Depending on the degree of line compensation the sub-synchronous torque components may or may not excite a particular torsional mode. Thus, SSR is a discrete event. Each torsional mode gets tuned only for a certain level of series compensation.
- Since the magnitude of the disturbance chosen is small, the super-synchronous frequency component in the torque is not excited appreciably and it damps out quickly.
- The example considers only the torsional interaction effects. In general, SSR is triggered due to the combined effects of torsional interaction and induction generator effects.
- It is not easy to identify a torsional mode from the time-domain plot of speed signals. For example, in Figure 11 the speed signal is dominated with two modes: torsional and swing mode. This task of identifying a mode is much more complex if the mechanical system offers a range of torsional frequencies as in the IEEE FBS. In such cases an evaluation of *modal speeds* from the actual speed variations offers a way to identify a mode [16].

3.0 CONCLUSIONS

1) In this paper a simple example is discussed which illustrates the various procedural aspects involved in SSR analysis.
2) The example presented clearly brings out the interactions between the electrical and mechanical systems through a systematic study of individual systems.
3) It is felt that this paper will be of great help to introduce SSR related issues to graduate students without much modelling details and motivates them to take up advanced studies with respect to the IEEE FBS where many intricate modal interaction exists.

ACKNOWLEDGEMENT

Authors would like to thank Prof. A. M. Kulkarni, EE Dept., IIT Bombay, for his valuable suggestions.

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