

Computed Lightning Electric Field Ratios of First and Subsequent Return Strokes

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Electromagnetic fields generated due to 'typical' first return stroke (FS) and subsequent lightning return stroke (SS) have been computed and compared. The simulation results are discussed keeping in view the field data reported in several recent literature, which compares the severity of first and subsequent return strokes. The MTLE based engineering model is adopted to compare the severity of lightning return strokes (FS/SS) as a function of radial distance and the worst case ground conductivity.

In general, the magnitude of electric field peak due to of first return stroke is nearly twice that of field peak due to subsequent return stroke as reported in the literature, based on the data collected by lightning detection, information and field measurement systems. The present simulation results not only substantiate this fact but also try and assess few parameters responsible for low ratio of FS/SS (reported in some cases) through simulation process; as causes for these low ratios are yet to find satisfactory explanation.

Keywords: *Electromagnetic fields, First return stroke, Ground conductivity, Lightning and Subsequent return stroke.*

1.0 INTRODUCTION

Knowledge of electromagnetic (EM) fields associated with lightning is essential to understand the indirect effects of lightning. These electromagnetic fields can result into induced over voltages in the objects they couple with, resulting in an indirect stroke. The threat due to lightning indirect strokes and the electromagnetic compatibility (EMC) studies depend on characteristics of both, the objects to which the lightning EM fields (LEMF) couple and the characteristics of lightning strokes. A typical cloud-to-ground (CG) flash will have one first return stroke and one or more subsequent return strokes. Based on the difference in their characteristics, typically, lightning strokes are grouped into two types; namely, first return

stroke (FS) and subsequent return strokes (SS). The subsequent return strokes can be more than one. The field observation reports indicate that, average number of subsequent return strokes can be 4–5 in a multiple stroke flash [1–4] of negative CG flashes. Also, 80% of CG lightning flashes consist of multiple strokes [1]. The two important parameters of these lightning stroke currents are the current peak, (I_{peak}) and maximum time rate of change of current, $(di/dt)_{\text{max}}$. The 'typical' first and subsequent return strokes of negative CG discharges are characterized by these parameters [5–6] with numeric values as given in Table 1.

These are the lightning research community accepted 'typical' first and subsequent return strokes (representative); widely discussed in the literature [1]. The lightning stroke current

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parameters decide the lightning field parameters, namely, peak and maximum rate of change of electric and magnetic fields. These parameters in turn are responsible for induced over voltages and their characteristics. In spite of the knowledge related to lightning current parameters one simple question of interest to all remains to be fully addressed is “first or subsequent, which event is more severe?” [7].

TABLE 1		
TYPICAL CHARACTERISTICS OF LIGHTNING RETURN STROKES [5]		
Type of stroke	Parameters	
	I_{peak}	Maximum di/dt
First return stroke (FS)	30 kA	12 kA/ μ s
Subsequent return stroke (SS)	12 kA	40 kA/ μ s

The relative magnitudes (in CG lightning flash) of the electric field peak of first and subsequent return stroke are important in comparing the severity of the strokes in a multiple-stroke flash. Such data have been examined and reported based on the lightning flashover data recorded in various countries all over the globe [8–13]. Such field measurement efforts using lightning detection, information and field measurement system along with lightning location systems (LLS) greatly add to the understanding of lightning phenomena from the point of view of stroke statistics and, their generalizations. Aiding such research efforts, through simulation process, has been a unique attempt of this present work.

The aim of the present work is to examine the relative magnitude of electric field peaks of ‘typical’ first and subsequent lightning return stroke through, modeling and simulation. Severity of FS vs SS is analyzed through simulation and compared with field measured data (reported in the literature) [8–13]. This simulation effort also attempt to bring out the effect of finite ground conductivity (worst case) on relative magnitude of electric field peaks. These simulation results seem to give some clue for low ratios of FS/SS observed and reported in some cases of field

measurements (as cause for low ratios are yet to be reasoned [8]). Before reporting the simulation results a brief review of FS/SS ratios (of measured values) found in the literature are summarized [8–13].

2.0 SUMMARY OF FS/SS RATIO

Usually a field recorded data, during lightning event, will have multiple flashes with each flash containing multiple strokes (on an average 4–5 strokes; in few cases it can be a single stroke flash). Each stroke has a different peak. This statistical data is analyzed by computing the averages and the method of analysis adopted can differ. In literature [8–13], the field measured data related to first and subsequent return stroke peaks (E-field) are analyzed by adopting three (one or more of) different methods. In evaluation and analysis, although arithmetic means (AM) are used, some of the researchers have tried to analyze by evaluating the geometric means (GM), in arriving at the ratio of FS/SS.

3.0 THE METHODS ADOPTED ARE:

Method A1: This accounts for many flashes of multi strokes. For each stroke order, average of the all the corresponding stroke order (sequential number of a stroke in a flash) magnitude (taken from all the flashes) is calculated and then the FS/SS ratios of these means are evaluated for each stroke order, and further the means of these FS/SS are evaluated (including single stroke flashes).

Method A2: This accounts for flashes of multi strokes only (excluding single stroke flashes). For each stroke order, average of all the corresponding stroke order (sequential number of a stroke in a flash) magnitude (taken from all the flashes) is calculated and then the FS/SS ratios of these means are evaluated for each stroke order, and further the means of these FS/SS are evaluated (excluding single stroke flashes).

Method B: The ratio of FS peak field to mean value of peak fields of all the SS in each multiple

stroke flash is calculated. Then, means of such FS/SS ratios of multiple flashes is evaluated.

One of the reasons for differences in the results reported from different researchers [8–13] is thought to be due to difference in methodology adopted in arriving at ratio FS/SS in analyzing the data [9]. A summary of the ratios of FS/SS E-field peaks reported in these related literature are brought out in the form of Tables and are given in Table 2–3. These are the reported results from various research groups from different countries, with field data pertaining to the lightning events in their country.

TABLE 2		
AVERAGE FS/SS E-FIELD RATIO AT DIFFERENT COUNTRIES SUMMARIZED FROM DISCUSSION OF REFERENCES [8–9]		
Country	FS/SS E-field ratio	
	A1	
	AM	GM
USA (Florida) [8–9]	2.1	1.7
Austria [9]	2.3	1.6
Brazil and Sweden [9]	2.4	1.9

TABLE 3						
SUMMARY OF AVERAGE FS/SS E-FIELD RATIO AT DIFFERENT COUNTRIES FOUND IN LITERATURE [10–13]						
Country	E-field ratio (FS/SS) using different methodology					
	A1		A2		B	
	AM	GM	AM	AM	GM	AM
Austria [10–11]	1.00	–	1.18	–	1.32	–
	1.04	–	1.41	–	1.60	–
Brazil [12]	1.64	1.69	1.69	1.75	1.78	1.53
Sweden [13]	1.70	1.60	2.00	1.80	2.10	1.81

Modern LLS with multiple stations not only help in locating the lightning location but also give a

peak current estimate for each stroke. The estimate is based on magnetic radiation field peaks and distances. In arriving at current estimates current peaks are assumed to be proportional to the field peaks [9]. Reference [9] discusses some of these results of FS/SS current ratios. The AM of such peaks is in the range of 1.6–2.1 [9]. Discussion relating to comparison of LLS systems can be found in references [14–15]. The effort there is to see the relative merit and accuracies of these detection systems being adopted in different countries.

In general, as observed from the Tables 2–3, AM of FS/SS ratio varies in the range 1–2.4. This wide variation (especially the lower values of FS/SS in Austria studies) has been the subject of discussion in the literature.

The topic has also given rise to a wide scope for further study and research in knowing the relative severity of first return stroke in relation with that of subsequent return stroke. With this in mind the present work has attempted to evaluate the FS/SS ratio with ‘typical’ first and subsequent strokes, through simulation process. In the present work, one of the most widely used, Modified Transmission Line with Exponential current decay (MTLE) model (originally proposed by C. A. Nucci) has been adopted [16] for computing E-fields. The specific details of the simulation process adopted in this work are described in the next section.

4.0 SIMULATION DETAILS

The modeling of lightning return strokes and then computation of electromagnetic fields due to them, in general, involve following two major steps [17]:

- Modeling and simulation of spatial-temporal distribution of lightning return stroke current, along the lightning channel.
- Calculating electromagnetic fields produced by making use of lightning return stroke current model, over perfectly conducting ground.

Further, for situation of finite ground conductivity, one can evaluate the electromagnetic fields over the surface of the ground and above the ground using Cooray-Rubinstein approximations [18–19].

These steps and the related simulation details (specific to present study) are briefly narrated in this section. The model formulations are based on (and are similar to) the details given in reference [17].

4.1 Return Stroke Current

For determining the electric and magnetic fields, it is necessary to know the return stroke current distribution along the channel. The lightning channel is assumed to be straight and vertical, above the ground plane (orthogonal to it), starting from the striking point at ground (at channel base). The geometry of the simulated lightning return stroke and the associated observation point above the perfectly conducting ground plane is as shown in Figure 1.

In MTLE model [16], the lightning current is allowed to decrease (exponentially) with the height, while propagating along the channel (upward). A current-element $i(dz')$ is chosen along the path. The infinite ground plane is simulated using an equivalent image current-element at $-z'$, below the ground plane. The observation point above the ground plane is at $P(r, \Phi, z)$; Where 'r' is the radial distance and 'z' is the height above the ground-plane.

The height 'H', of the cloud above the ground plane is assumed to be 8 km. The current through lightning channel, in the MTLE model [16] adopted from reference [20] is given by the Eqn. (1)

$$i(z', t) = u(t-z'/v) e^{-(z'/\lambda)} i(0, t-z'/v) \quad \dots (1)$$

where 'v' is the velocity of the return stroke, ' λ ' is a current decay constant, $u(t)$ is the Heaviside function and $i(0, t)$ is the current at ground.

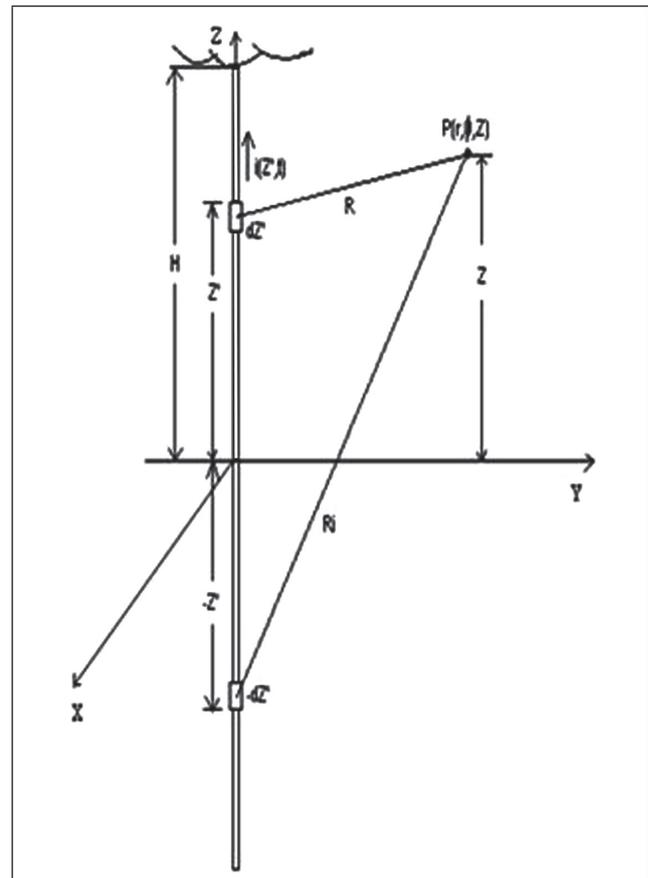


FIG. 1 GEOMETRICAL DETAILS OF LIGHTNING CHANNEL USED FOR LIGHTNING ELECTROMAGNETIC FIELD CALCULATIONS

Using this model for the simulation of lightning stroke currents, the electromagnetic fields surrounding the lightning discharge are calculated; as discussed below.

4.2 Computation of Electromagnetic Fields

Horizontal and vertical component of the electric fields, azimuth component of magnetic field (magnetic fields not reported and discussed in this paper) due to an elemental dipole of current $I(z', t)$, for an infinitesimal lightning channel length dz' at a height z' from the ground are calculated at an observation point 'P'. This is accomplished for a perfectly conducting ground, by adopting the expressions given in reference [17]. A simulation code for the purpose is developed in MATLAB [21]. Because of cylindrical symmetry of the problem, the electromagnetic fields at any point are obtained (with ease using cylindrical coordinate system) with the return stroke channel placed along the z-axis. The total field at the

observation point is obtained by integrating over the length of the current channel [16, 22] (also accounting for its image).

4.3 Electromagnetic Fields With Finite Conductivity Ground

In the case of finite conductivity ground, the horizontal component of the electric field gets altered. For a finitely conducting ground the horizontal electric field is computed using cooray-rubinstein (CR) approximation [18–19], known as CR approximation. In CR approximation the horizontal electric field is computed by adding a term to electric field values obtained for infinite ground. The term added is obtained from surface impedance calculations [18].

4.4 Parameters Used in the EM-Field Calculation

Calculations of electromagnetic fields, using the code implemented are carried out for a typical observation point at a height, z of 10 m above the ground plane and at radial distances of 500 m to 100 km from the lightning channel (in discrete steps). ‘Typical’, first return and subsequent return strokes (characterized by its important lightning current parameters as given in Table 1 [5]) are adopted for the simulations. The worst case finite ground conductivity of 0.0001 S/m is used to simulate the scenario of finite ground condition. The return stroke velocity of the lightning current used in the present simulation are 130 m/ μ s (FS) and 190 m/ μ s (SS) (The typical range of return stroke velocity, as stated in the literature is $\frac{1}{3} C$ to $\frac{2}{3} C$; where ‘C’ is the velocity of light [23]). The decay constant ‘ λ ’, (in MTLE return stroke model) is assumed to be equal to 2 km (The typical range of decay constant as stated in the literature is 1–2 km [24]).

5.0 RESULTS AND DISCUSSION

The simulation results of lightning electric fields for the observation point ($z = 10$ m and $r = 5000$ m as a typical result) above the perfect ($\sigma = \infty$ m/s) and finitely ($\sigma = 0.0001$ m/s) conducting ground are presented below.

In general, the total E-field has three components [5], of which, induction and radiation fields are combined and plotted, as they contribute to the indirect effect of lightning stroke. Third component, namely the static field, is separated (and not shown). At very close distances of 100 m or less, both static and induction field components add to the radiation component. Beyond 100 m and up to 1000 m both induction and radiation fields contribute to the peak. Beyond 1000 m the fields are solely due the radiation component [17]. Although the contributions of the induction and radiation fields look smaller (in magnitude and duration), they are of importance from the point of indirect lightning influences. The induced voltages in the components illuminated by lightning can be attributed to induction and radiation components [17]. In the region of interest (of present study; 500 m–100 km) both induction and radiation components are computed for ‘typical’ first and subsequent lightning strokes (Table 1). To validate the code implemented in the present work, horizontal electric field results available in the literature for subsequent return stroke given in reference [25] are simulated with identical conditions. The relevant results of [25] have been successfully reproduced. Further the same code is used for numerical experimentation discussed in the subsequent sections.

5.1 Perfectly Conducting Ground

Figures 2–3 show the variation of vertical and horizontal component of electric field corresponding to first and subsequent strokes, respectively, above the perfect ground at a typical observation point ($z = 10$ m and $r = 5000$ m). Code implemented can also be used to compute the variation of magnetic (azimuth component) field at this (or any) observation point. Although, these magnetic field results, not being relevant to the present theme is not discussed.

The computed electric field results presented in Figures 2–3 are in good agreement with those of reference [5], as far as the trend of plots is concerned. The important observation based on the peaks of these plots is that, field peak is larger

for first return stroke (nearly 2 times) than that of subsequent return stroke. In general, these simulation results of ratio FS/SS compare well with those of field measured data that are reported in the literature [8–13].

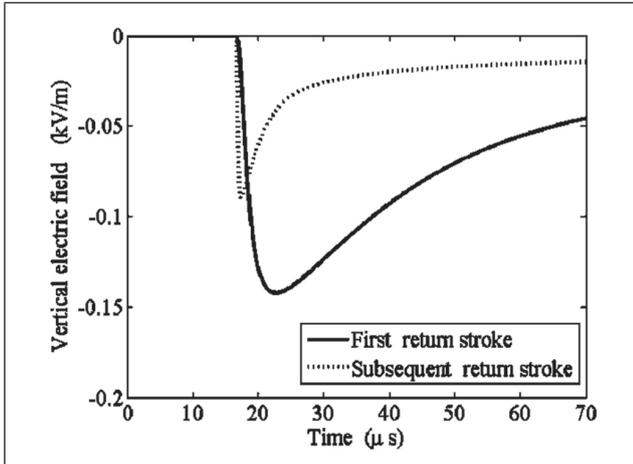


FIG. 2 VERTICAL COMPONENT OF ELECTRIC FIELD DUE TO FIRST AND SUBSEQUENT RETURN STROKE ABOVE THE PERFECTLY CONDUCTING GROUND AT $Z=10$ m, $R=5000$ m.

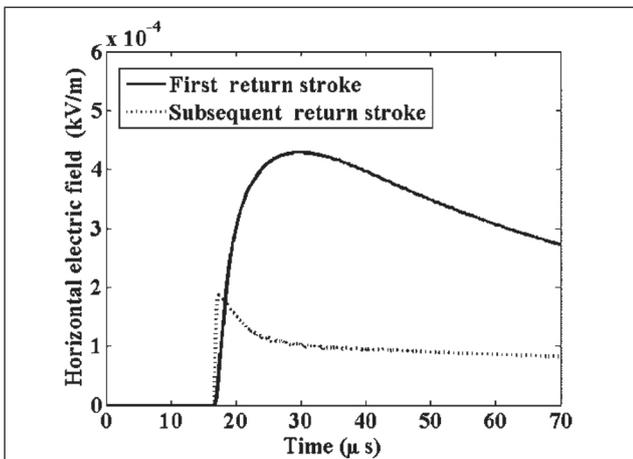


FIG. 3 HORIZONTAL COMPONENT OF ELECTRIC FIELD DUE TO FIRST AND SUBSEQUENT RETURN STROKE ABOVE THE PERFECTLY CONDUCTING GROUND AT $Z=0$ m, $R=5000$ m.

The purpose of this work being solely to compare the severity of typical first return stroke to that of typical subsequent, through simulation, the ratio of FS/SS at different radial distance are computed. Both horizontal and total electric field ratios of FS/SS thus computed are as given in Figure 4.

The computed results for plotting the Figure 4 are obtained using the simulation model based

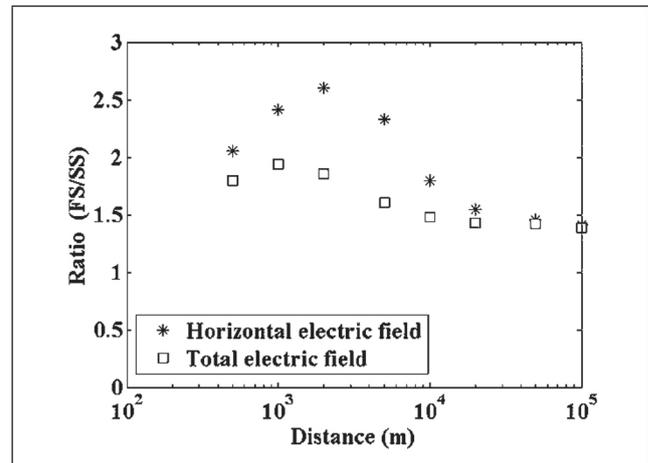


FIG. 4 RATIO OF FIRST TO SUBSEQUENT RETURN STROKE ELECTRIC FIELD PEAKS AS FUNCTION OF DISTANCE FOR THE OBSERVATION POINTS AT A HEIGHT OF $Z=10$ m, ABOVE THE PERFECTLY CONDUCTING GROUND.

on MTLE; to that extent these results are specific (although similar trends can be expected with other models). The ratios of ‘vertical electric field’ are nearly equal to that of corresponding ‘total electric field’ as the contribution of horizontal component in ‘total electric field’ is quite small. Despite the fact, it is the horizontal component which is of importance in calculating the induced voltages due to field coupling with transmission lines [26]. These ratios of FS/SS obtained by simulations are lower for the total (and vertical) electric field compared to the horizontal electric fields. The FS/SS ratio for horizontal electric field is in the range of 1.5–2.5 (and is a nonlinear function of distance). The FS/SS ratio for the vertical (and total) electric field is in the range of 1.5–2. Electric fields being a nonlinear function of distance, these ratios are spread over a range. The field measurement results of ratio FS/SS as discussed in the literature [8–13] (summarized in Tables 2–3) can be compared with those obtained via simulation. In general, the simulation results compare well with the global average of experimental measurements (which are due to the observations from the actual lightning environment) as discussed in the reference [9]. Reference [9] mentions about the some discrepancies between first and subsequent return stroke intensities reported from different studies.

Literature [26] indicates that, the horizontal component is much smaller compared to the vertical component. Similar observations of “smaller horizontal components” could be made through present simulations. It is the horizontal component of the electric field, which is affected by the ground conductivity, although its contribution to the total electric field is smaller. Keeping in view the importance of horizontal component [26], their ratios of FS/SS magnitudes, along with the total electric fields are presented in Figure 4. At any given distance FS/SS ratio of horizontal component is higher than the total electric field; in both the cases ratio is greater than unity.

5.2 Finitely Conducting Ground

The simulation results showing variation of horizontal electric fields, above a finitely conducting ground (0.0001 S/m) are presented in Figure 5, for a typical observation point ($z = 10$ m and $r = 5000$ m). In this Figure horizontal electric field due to ‘typical’ first and subsequent return strokes are compared. From the typical observation point simulation results given in Figure 5, it is seen that variation of horizontal electric fields are bipolar in nature which is in agreement with what has been reported in the literature [27], for finite ground conductivity situation.

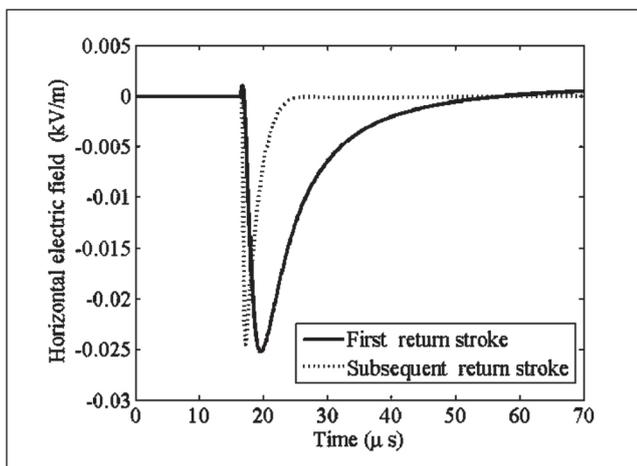


FIG. 5 HORIZONTAL ELECTRIC FIELD DUE TO FIRST AND SUBSEQUENT RETURN STROKE ABOVE GROUND AT $z = 10$ m, $r = 5000$ m FOR FINITELY ($\Sigma_G = 0.0001$ S/M) CONDUCTING GROUND.

The plot of ratio of peaks of first to subsequent strokes (FS/SS), obtained using the simulation code (developed) is as given in Figure 6.

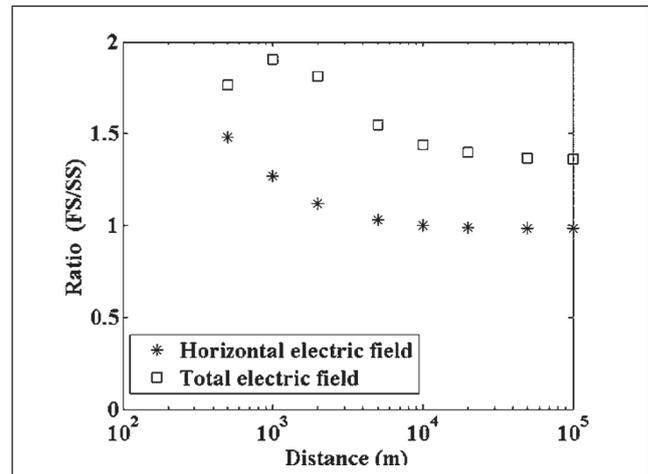


FIG. 6 RATIO OF FIRST TO SUBSEQUENT RETURN STROKE ELECTRIC FIELD PEAKS AS A FUNCTION OF RADIAL DISTANCE (FOR THE OBSERVATION POINTS AT A HEIGHT OF $z = 10$ m), ABOVE THE GROUND PLANE OF WORST CASE FINITE GROUND CONDUCTIVITY ($\Sigma_G = 0.0001$ S/M).

Figure 6 compares FS/SS ratio for total electric field with that FS/SS ratio of horizontal component of electric field. It is the ratio FS/SS of horizontal component of electric field which is affected by the ground conductivity and this ratio is lower than that of total electric field, unlike what is seen in Figure 4. The FS/SS ratio for horizontal electric field is in the range of 1.0–1.5 (and is a nonlinear function of distance) with the worst case ground conductivity; unlike for perfect ground situation, for which it is in the range of 1.5–2.5.

The effect of change in ground conductivity on first and subsequent return stroke can be studied using the simulation results given in Figures 3 and 5. They aid in comparing the horizontal electric fields. The field peak (induction and radiation components put together) in case of finite ground situation is higher in magnitude when compared with infinite ground conductivity. Even in the case of subsequent return strokes, the field peak (induction and radiation components put together) is higher for finite grounds (see Figures 3 and 5).

One of the basic inferences through present simulation work is that, the ground conductivity will affect the ratio of FS/SS (of the horizontal component of the field). Hence terrains can

influence and play a major role as far as the severity of return strokes is concerned.

6.0 CONCLUSIONS

Lightning return strokes severities of first and subsequent strokes (with 'typical' ones) are compared through the simulation process (probably for the first time); and it is observed that:

- The field magnitudes due first return stroke are higher compared to subsequent return stroke (ratio FS/SS > 1). This is true for both perfect and finitely conducting grounds.
- Ratio FS/SS obtained (by simulation) match fairly well with the literature reported FS/SS data (global average).
- Horizontal component of electric field due to lightning being of importance, the severity of FS and SS are compared (by computing) exclusively for this component, and are reported.
- The ratio of FS/SS of horizontal electric field is lowered due to decrease in ground conductivity (when compared to infinite ground conductivity situation).

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